Graz University of Technology

Institute of Geometry

Advanced Topics in Discrete Mathematics Seminar

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Seminarraum 2, Kopernikusgasse 24/IV.

The excedance quotient of the Bruhat order, Quasisymmetric Varieties and Temperley-Lieb algebras

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Let $R_n = \mathbb{Q}[x_1, x_2, \ldots, x_n]$ be the ring of polynomials in n variables and consider the ideal $\langle \mathrm{QSym}_n^+ \rangle \subseteq R_n$ generated by quasisymmetric polynomials without constant term. It was shown by J. C. Aval, F. Bergeron and N. Bergeron that $\dim\left(R_n / \langle \mathrm{QSym}_n^+ \rangle\right) = C_n$ the nth Catalan number. In the present work, we explain this phenomenon by defining a set of permutations QSV_n with the following properties: first, QSV_n is a basis of the Temperley–Lieb algebra $\mathsf{TL}_n(2)$, and second, when considering QSV_n as a collection of points in $\mathbb{Q}^n$, the top-degree homogeneous component of the vanishing ideal $\mathbf{I}(\mathrm{QSV}_n)$ is $\langle \mathrm{QSym}_n^+ \rangle$.

Our construction has a few byproducts which are independently noteworthy. We define an equivalence relation $\sim$ on the symmetric group S_n using weak excedances and show that its equivalence classes are naturally indexed by noncrossing partitions. Each equivalence class is an interval in the Bruhat order between an element of QSV_n and a 321-avoiding permutation. Furthermore, the Bruhat order induces a well-defined order on $S_n/\sim$. Finally, we show that any section of the quotient $S_n/\sim$ gives an (often novel) basis for $\mathsf{TL}_n(2)$.

This talk is based on joint work with Lucas Gagnon.

Cesar Ceballos