MATH 1300 A, Fall 2013
Solution Quiz 6

1. (*100 points*) A spherical balloon is expanding in such a way that its volume is increasing at 6 cubic centimeters per second. Find the rate of increase of the surface area of the balloon when its volume is 36π cm^3.

The equations for the volume and the surface area of a sphere in terms of the radius are

$$V = \frac{4}{3}\pi R^3 \qquad\qquad A = 4\pi R^2$$

We are given that

$$\frac{dV}{dt} = 6 \text{ cm}^3/\text{sec}$$

and we are ask to find $\frac{dA}{dt}$ when the volume is

$$V = \frac{4}{3}\pi R^3 = 36\pi \text{ cm}^3.$$

Solving this equation we find the radius of the balloon:

$$R^3 = \frac{(3)(36\pi)}{4\pi} = 27 \quad \Rightarrow \quad R = 3 \text{ cm}$$

In order find to rate of increase of the surface area we apply the chain rule:

$$\frac{dA}{dt} = \frac{dA}{dR}\cdot\frac{dR}{dV}\cdot\frac{dV}{dt} = \frac{\frac{dA}{dR}\cdot\frac{dV}{dt}}{\frac{dV}{dR}} = \frac{8\pi R\cdot 6}{4\pi R^2} = \frac{12}{R} = \frac{12}{3} = 4 \text{ cm}^2/\text{sec}$$