MATH 1300 A, Summer 2015 Name: ______________________

Quiz 1 Student number: ______________________

Solution

1. (*70 points*) Consider the function $f(x) = \frac{4x+3}{2x-6}$.

(a) Is f one-to-one? f is one to one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

$$\frac{4x_1+3}{2x_1-6} = \frac{4x_2+3}{2x_2-6}$$

$$(4x_1+3)(2x_2-6) = (4x_2+3)(2x_1-6)$$

$$\cancel{8x_1x_2} - 24x_1 + 6x_2 - \cancel{18} = \cancel{8x_2x_1} - 24x_2 + 6x_1 - \cancel{18}$$

$$30x_2 = 30x_1$$

$$x_2 = x_1$$

$\Rightarrow$ Yes, f is one-to-one

(b) Find a formula for the inverse of f.

$$y = \frac{4x+3}{2x-6} \Rightarrow y(2x-6) = 4x+3$$

$$2yx - 6y = 4x+3$$

$$2yx - 4x = 6y+3$$

$$(2y-4)x = 6y+3$$

$$x = \frac{6y+3}{2y-4}$$

replacing y's by x's.

$$\boxed{f^{-1}(x) = \frac{6x+3}{2x-4}}$$

(c) Find the domain and the range of f.

domain $f = (-\infty, 3) \cup (3, \infty)$ because $2x-6 \neq 0 \Rightarrow x \neq 3$

range f = domain $f^{-1} = (-\infty, 2) \cup (2, \infty)$ because $2x-4 \neq 0 \Rightarrow x \neq 2$

2. (*30 points*) Mark true or false. No justification is needed.

(a) If $f(x) = x^2$ and $g(x) = x+1$, then $(f \circ g)(x) = x^2 + 1$.

FALSE : $(f \circ g)(x) = f(g(x)) = (x+1)^2$

(b) $\arcsin(\sin(x)) = x$.

FALSE : For example if $x = \pi/2 + 2\pi$, $\arcsin(\sin(\pi/2 + 2\pi)) = \pi/2 \neq x$

(c) $\tan(\frac{-5\pi}{6}) = \sqrt{3}$.

FALSE : $\tan(\frac{-5\pi}{6}) = \tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$