Cesar Ceballos
Institute of Geometry
TU Graz

Combinatorial Reciprocity Theorems via Geometry
Exercise Sheet 01

Winter Semester
2020-2021

Exercise 1

Find the chromatic polynomial of the following graphs:

  (i) a path with k nodes.

 (ii) a cycle with k nodes.

(iii) a complete graph with k nodes.

Exercise 2

Find the number of acyclic orientations of a wheel with k spokes.

For example, the wheel with 8 spokes is:

Exercise 3

Find the number $\widetilde{\chi}_G(n)$ of compatible pairs (ρ, c) of an acyclic orientation ρ and an n-coloring c of G of the following graphs:

  (i) a triangle

 (ii) a complete graph with k nodes.

Verify that $\widetilde{\chi}_G(n) = (-1)^k \chi_G(-n)$, where $\chi_G(n)$ is the chromatic polynomial of G.

Exercise 4

For a pair of natural numbers $k, n \in \mathbb{N}$ where k is fixed, define the function

$$f_k(n) := 1^k + 2^k + \cdots + n^k.$$

Show the following properties:

  (i) $f_k(n)$ agrees with the evaluation of a polynomial in n.

 (ii) the evaluation of this polynomial at negative numbers is also a sum of kth powers:

$$f_k(-n) = (-1)^{k+1} f_k(n-1).$$