Exercise 1

Show that if a graph G has a bridge then G has no nowhere zero \mathbb{Z}_n -flow, that is, the flow counting function $\varphi_G(n) = 0$.

Exercise 2

Find a formula for the flow counting function $\varphi_G(n)$ of the following graphs:

- (i) a cycle with k nodes.
- (ii) the graph consisting of two vertices and k (multiple) edges connecting them.
- (iii) a complete graph with 4 nodes.

Exercise 3

Let G = (V, E) be a bridgeless connected graph. Show that $\varphi_G(n)$ agrees with a polynomial with integer coefficients of degree |E| - |V| + 1 and leading coefficient 1.

Hint: Use a deletion-contraction argument.

Exercise 4

Show that the Petersen graph has no nowhere \mathbb{Z}_4 -flow. Find a formula for its flow polynomial $\varphi_G(n)$.

