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Combinatorial Reciprocity Theorems via Geometry
Exercise Sheet 03

Winter Semester
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Exercise 1

Let Π be a finite poset. Show that the counting function

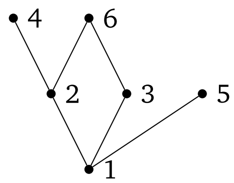
$$\Omega_\Pi(n) := |\{\varphi : \Pi \to [n] \text{ order preserving map}\}|$$

agrees with a polynomial of degree $|\Pi|$ with rational coefficients (the *order polynomial*).

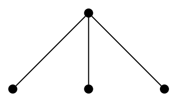
Exercise 2

Find the strict order polynomial $\Omega^\circ_\Pi(n)$ of the following posets.

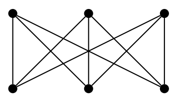
(i) the poset D_6 consisting of the elements $[6]$ ordered by divisibility:



(ii) the poset whose Hasse diagram is a tree consisting of a root with k children. For example, for $k = 3$:



(iii) the poset whose Hasse diagram is the complete bipartite graph $K_{3,3}$:



Exercise 3

Find the order polynomial $\Omega_\Pi(n)$ of the graphs in the previous exercise. Verify that the following combinatorial reciprocity holds:

$$(-1)^{|\Pi|}\Omega^\circ_\Pi(-n) = \Omega_\Pi(n).$$