Exercise 1

Let $\mathcal{S} = \operatorname{conv}\left\{\binom{a_1}{b_1}, \binom{a_2}{b_2}\right\}$, with $a_1, a_2, b_1, b_2 \in \mathbb{Z}$ be a *lattice segment*.

(i) Show that

$$\operatorname{ehr}_{\mathcal{S}}(n) = Ln + 1$$

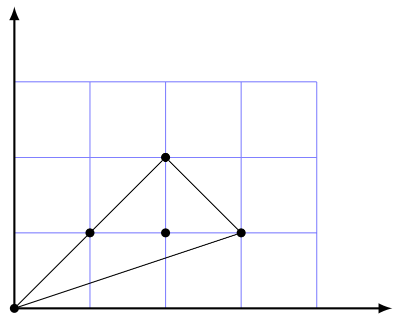
where $L = |\gcd(a_2 - a_1, b_2 - b_1)|$ is the *lattice length* of $\mathcal{S}$.

(ii) Show that

$$(-1)^{\dim \mathcal{S}} \operatorname{ehr}_{\mathcal{S}}(-n) = \operatorname{ehr}_{\mathcal{S}^\circ}(n).$$

Exercise 2

Let $\Delta = \operatorname{conv}\left\{\binom{0}{0}, \binom{3}{1}, \binom{2}{2}\right\}$. Find the Ehrhart polynomials $\operatorname{ehr}_\Delta(n)$ and $\operatorname{ehr}_{\Delta^\circ}(n)$.



Exercise 3

Let $\mathbf{v}_1 = \binom{a}{b}$ and $\mathbf{v}_2 = \binom{c}{d}$ be two vectors in $\mathbb{Z}^2$, and let Q be the half open parallelogram

$$Q := \{\lambda \mathbf{v}_1 + \mu \mathbf{v}_2 : 0 \leq \lambda, \mu < 1\}.$$

Show (for example, by tiling the plane by translates of Q) that

$$\operatorname{ehr}_Q(n) = An^2,$$

where $A = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$.

Exercise 4

Let $P \subset \mathbb{R}^2$ be a lattice polygon with area A, I interior integer points, and B integer points on the boundary. Show that:

(i) $A = I + \frac{1}{2}B - 1$.

(ii) $\operatorname{ehr}_{P^\circ}(n) = An^2 - \frac{1}{2}Bn + 1$.

(iii) $\operatorname{ehr}_P(n) = An^2 + \frac{1}{2}Bn + 1$.