## Exercise 1

Let $\Pi$ be the poset consisting of $d$ pairwise incomparable elements (an antichain).

(i) Show that the lattice $\mathcal{J}(\Pi)$ of order ideals of $\Pi$ is the Boolean lattice $B_d$.

(ii) Define the zeta polynomial $Z_{B_d}(n) := \zeta_{B_d}^n(\hat{0}, \hat{1})$ where $\zeta_{B_d}$ is the zeta function of $B_d$. Show that

$$Z_{B_d}(n) = n^d.$$

(iii) Verify that

$$\Omega_\Pi(n) = \zeta_{\mathcal{J}(\Pi)}^n(\emptyset, \Pi).$$

## Exercise 2

Let $\Pi = [k]$.

(i) Show that $\mathcal{J}(\Pi) \cong [k+1]$.

(ii) Show that the zeta polynomial $Z_{[k+1]}(n) := \zeta_{[k+1]}^n(\hat{0}, \hat{1})$ is

$$Z_{[k+1]}(n) = \binom{n+k-1}{k}.$$

(iii) Verify that

$$\Omega_\Pi(n) = \zeta_{\mathcal{J}(\Pi)}^n(\emptyset, \Pi).$$

## Exercise 3

Let $\Pi$ be a finite poset and $I(\Pi)$ the incident algebra of $\Pi$. Show that $\alpha \in I(\Pi)$ is invertible if and only if $\alpha(x, x) \neq 0$ for all $x \in \Pi$.