Exercise 1

Given linearly independent vectors $v_1, \dots, v_k \in \mathbb{R}^d$ consider the cone $C = \operatorname{cone}\{v_1, \dots, v_k\}$ and the half open parallelepipeds

$$\check{\square} = [0,1)v_1 + \cdots + [0,1)v_k,$$
$$\hat{\square} = (0,1]v_1 + \cdots + (0,1]v_k.$$

For $k = 2$ and the vectors $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, and $v_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$:

(i) Find the integer point transforms $\sigma_{\check{\square}}(z)$ and $\sigma_{\hat{\square}}(z)$.

(ii) Find the integer point transforms $\sigma_C(z)$ and $\sigma_{C^\circ}(z)$.

(iii) Verify that $\sigma_C(\frac{1}{z}) = (-1)^k \sigma_{C^\circ}(z)$.

Exercise 2

Let $C = \operatorname{cone}\{v_1, v_2, v_3\}$ and $\square = [0,1)v_1 + [0,1)v_2 + [0,1)v_3$ be the cone and the fundamental parallelepiped spanned by

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}.$$

(i) Which integer points are in $\square$? that is, find $\square \cap \mathbb{Z}^3$.

(ii) Find the Hilbert series $H_C^a(z)$ of C for the grading $a = (0,0,1)$.

Exercise 3

Repeat the previous exercise for

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

Exercise 4

Find the Ehrhart series $\operatorname{Ehr}_P(z)$, and use it to find the Ehrhart polynomial $\operatorname{ehr}_P(n)$, of the following polytopes:

(i) $P = \operatorname{conv}\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right\}$.

(ii) $P = \operatorname{conv}\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$.