Variational Shape Reconstruction

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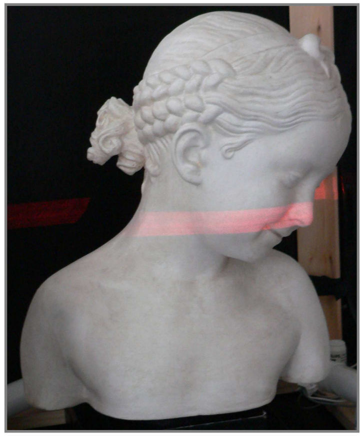
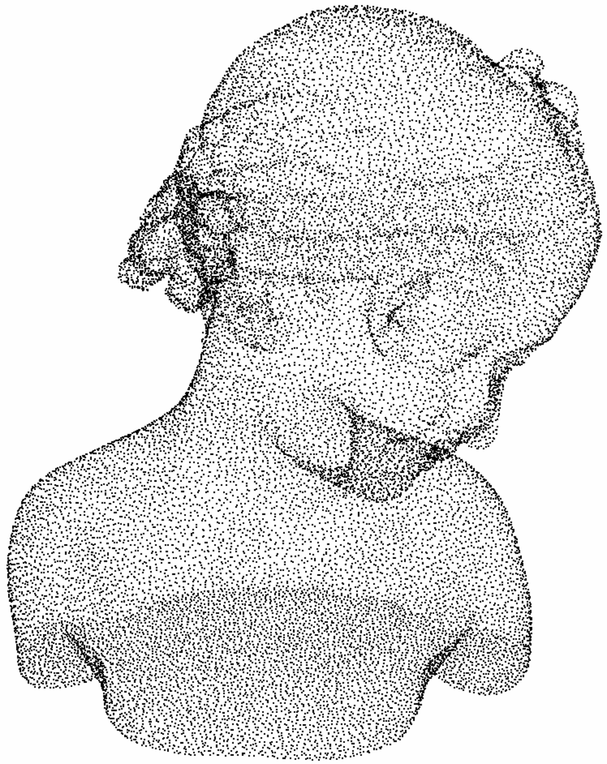
Motivations

- Surface reconstruction from point sets:
  - Unorganized
  - Unoriented (no oriented normals)
  - Non-uniform & sparse sampling
  - Possibly with noise
    - uncertainty in measurements
    - registration noise

Previous Work

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Early work:

[Boissonnat 84, Hoppe et al. 92, Curless-Levoy 96]

Delaunay-based

(Crust, Powercrust, Cocone, Tight cocone, …) [Amenta, Bern, Choi, Dey, Kolluri, Goswami, Giesen]

Deformable models

[Esteve, Sharf]

Spectral approach (normalized cuts) [Kolluri- Shewchuk-O'Brien]

Implicit surfaces:

- RBFs [Carr, Turk, Belyaev, Ohtake, Schlick]
- MLS [Levin, Alexa, Amenta, Kil]
- Poisson reconstruction [Kazhdan-Bolitho-Hoppe-Burns]

Graph-based (min-cuts)

[Boykov-Kolmogorov, Vogiatzis, Paris, Hornung-Kobbelt]

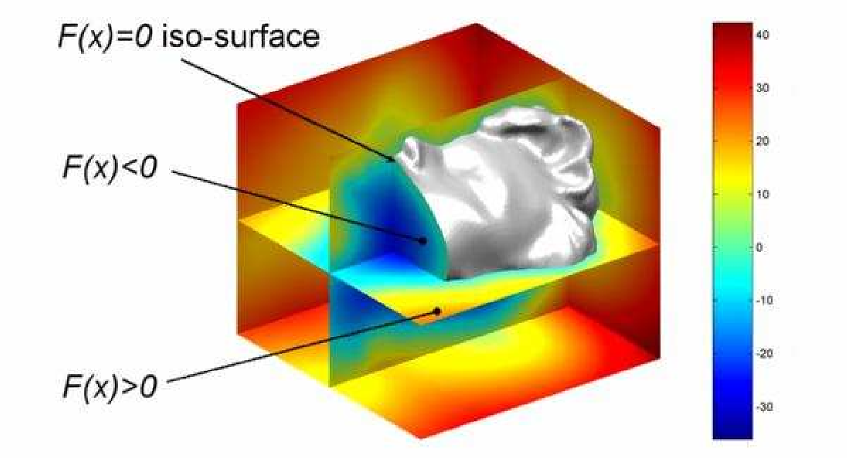
Delaunay-based

- Several Delaunay algorithms are **provably correct…**

Delaunay-based

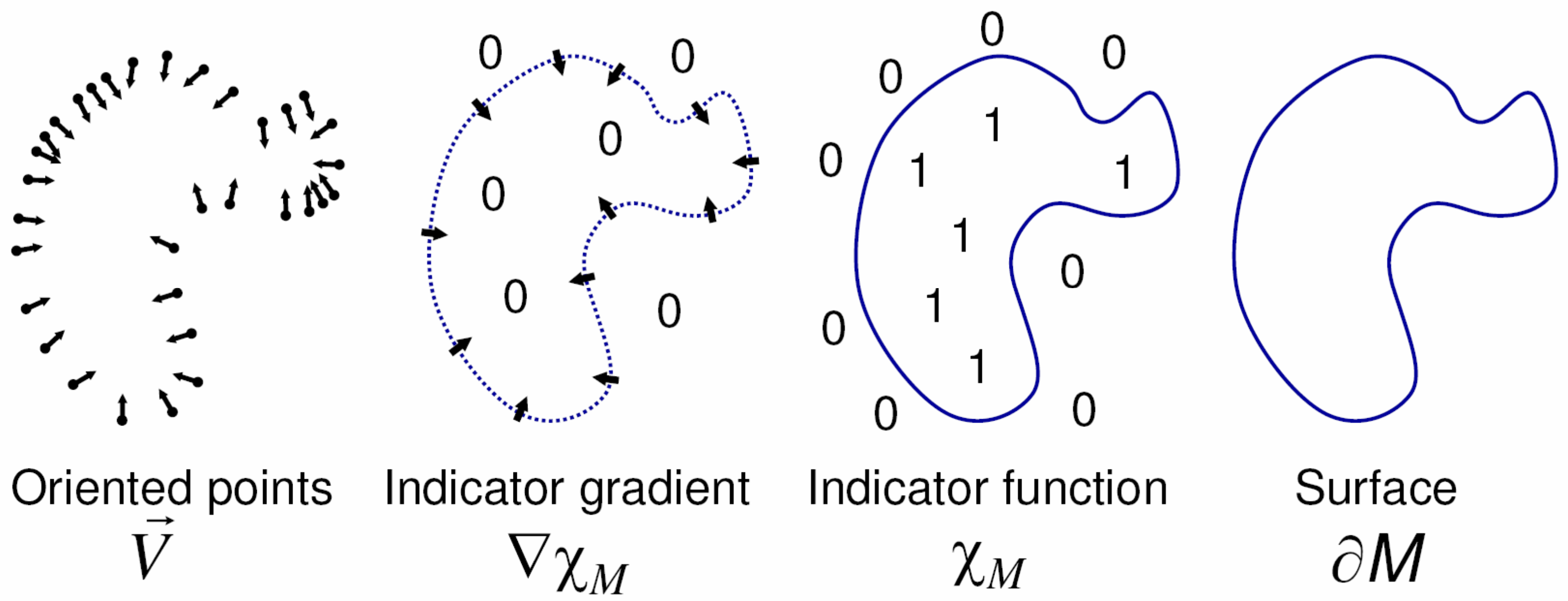
- Several Delaunay algorithms are **provably correct…** in the absence of noise and undersampling.

- Motivates reconstruction by fitting **approximating** implicit surfaces



Poisson Reconstruction

- Find an implicit function such as its **gradient best fits** a set of oriented normals



[Kazhdan-Bolitho-Hoppe 06]

Poisson Reconstruction

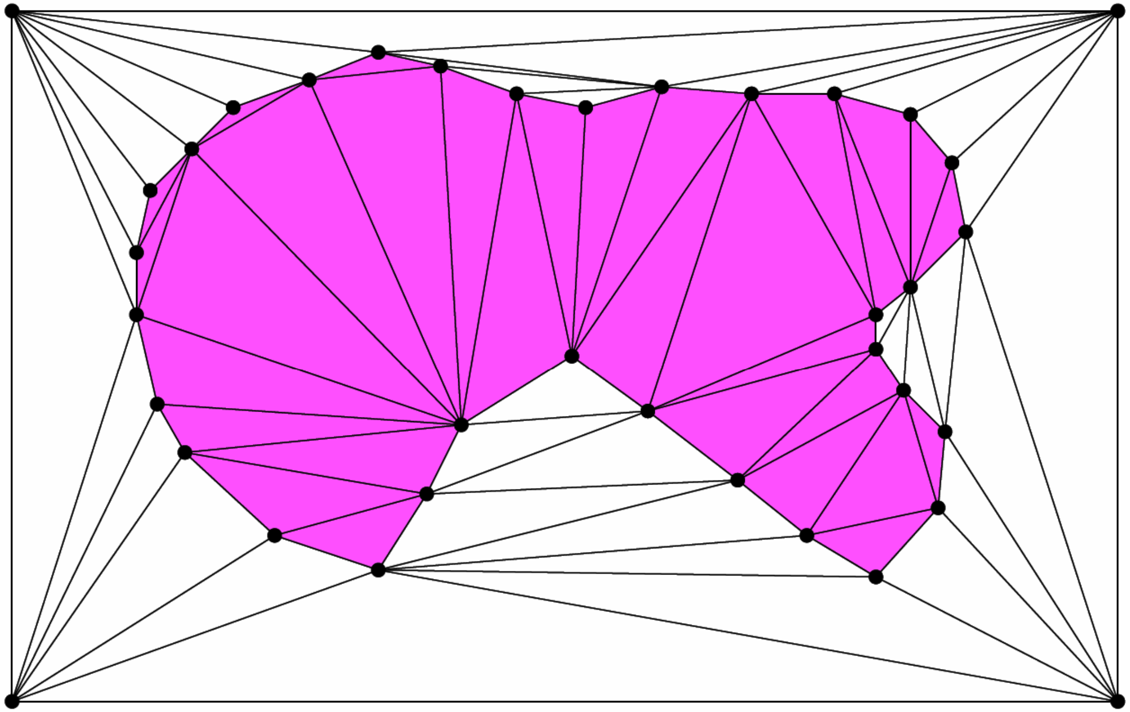
- **Orienting the normals**
  - ill-posed problem?

Poisson Reconstruction

- **Orienting the normals**
  - ill-posed problem?
  - basically as hard as the reconstruction problem.

Spectral Surface Reconstruction

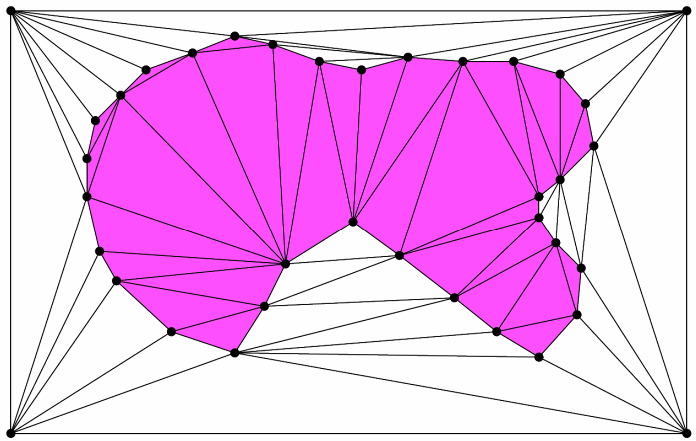
- Use **spectral graph partitioning** to decide which Delaunay tetrahedra are inside/outside the object.



[Kolluri-Shewchuk-O'Brien 04]

Spectral Surface Reconstruction

- Use **spectral graph partitioning** to decide which Delaunay tetrahedra are inside/outside the object.
  + ***global*** *approach (eigenvalue problem)*
  - ***interpolate*** *subset of data points*
  - no control over smoothness



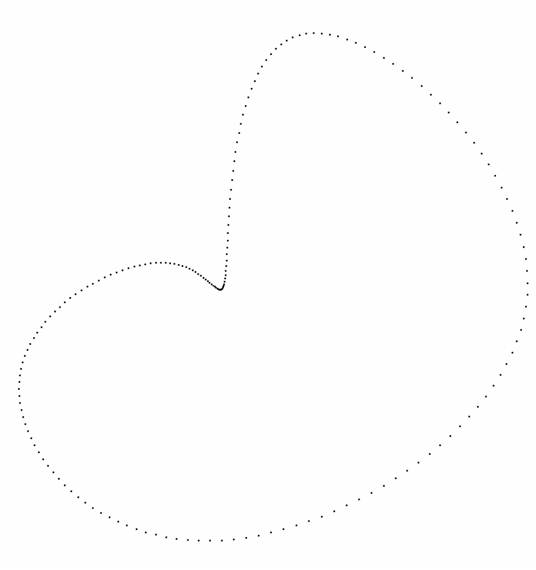
[Kolluri-Shewchuk-O'Brien 04]

Our Approach

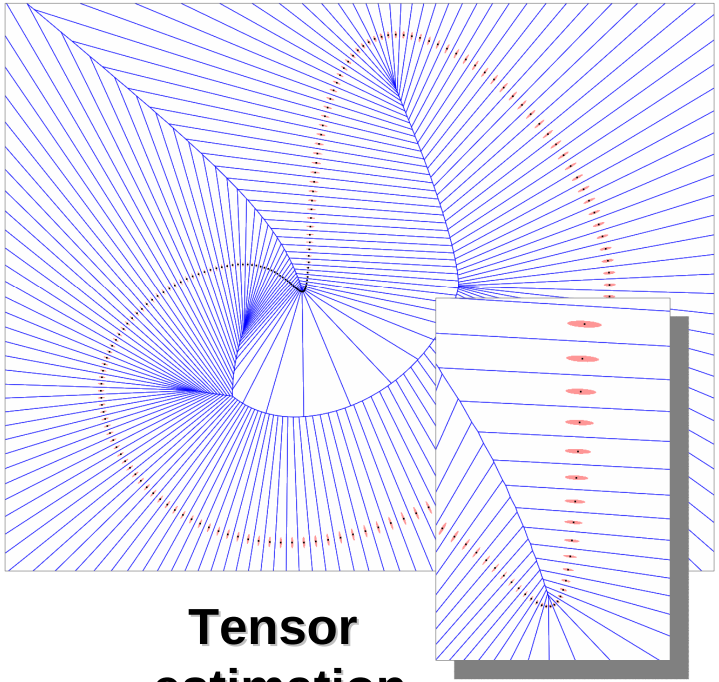
- **Delegate normal orientation** to fitting stage
  - Fit normal ***directions***
  - ***Reliability*** of directions can be used as well
- Offer control of **surface smoothness**
  - Trades fitting for smoothness

- **Output**:
  - Watertight
  - Approximating
  - Automatically adapted to sampling quality

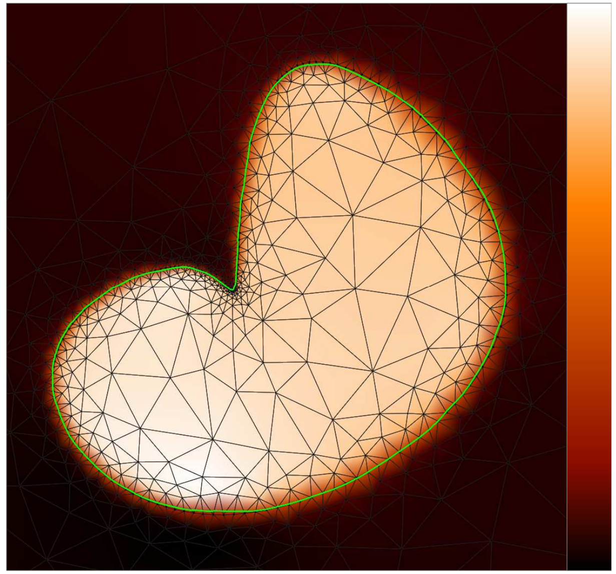
Overview



Point set

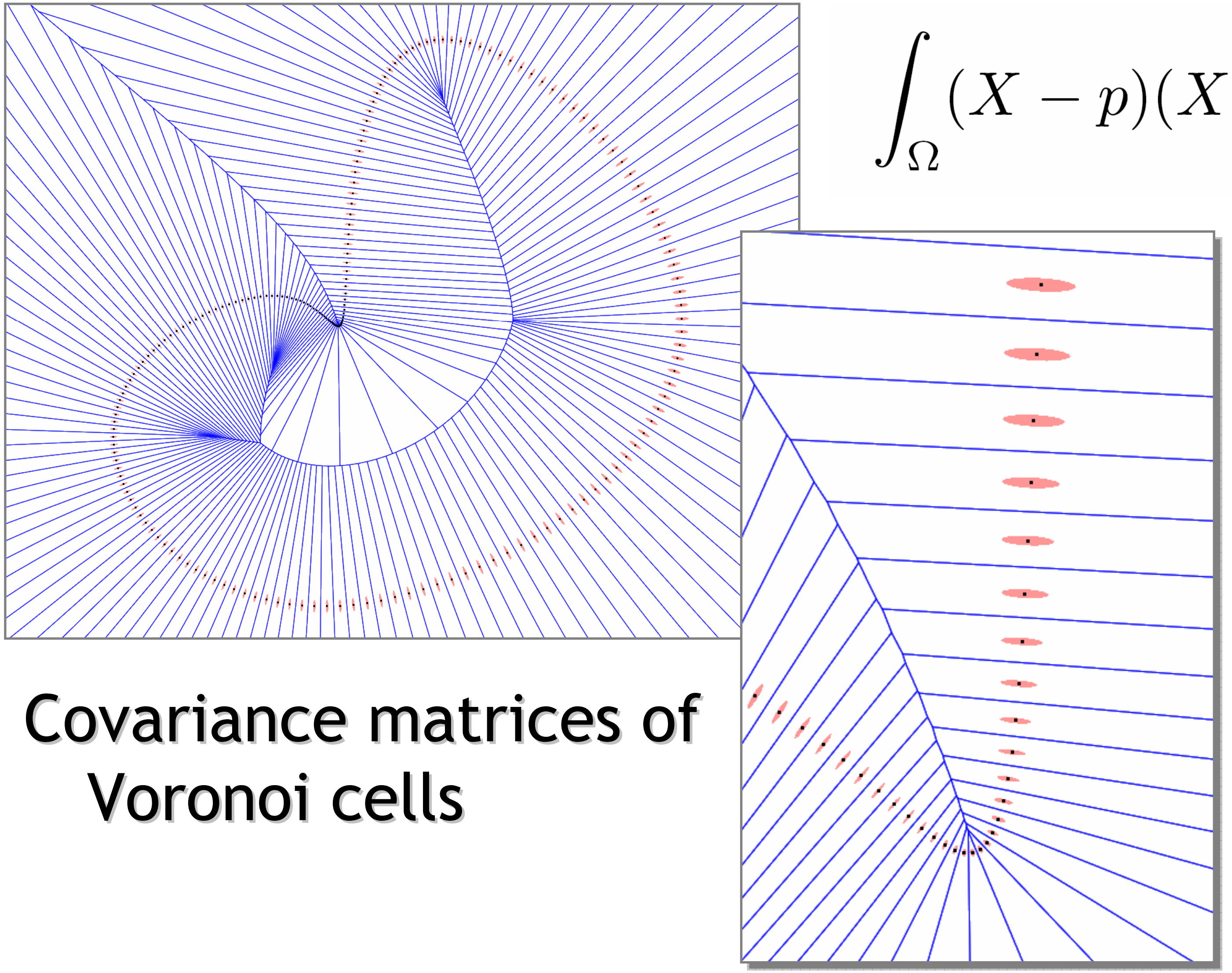


Tensor estimation



Implicit function + contouring

Tensor Estimation

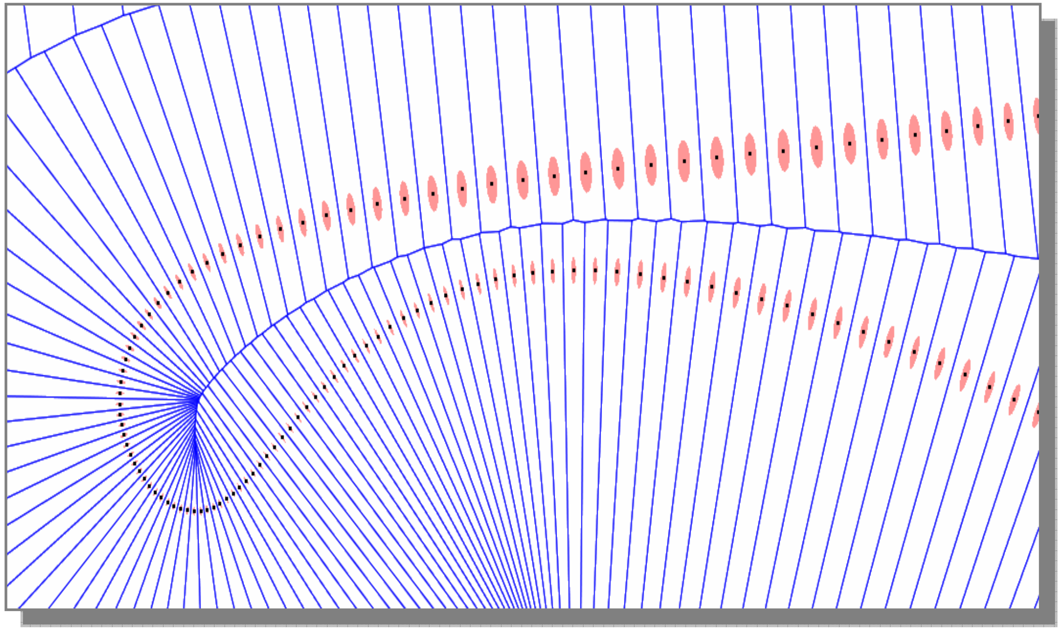


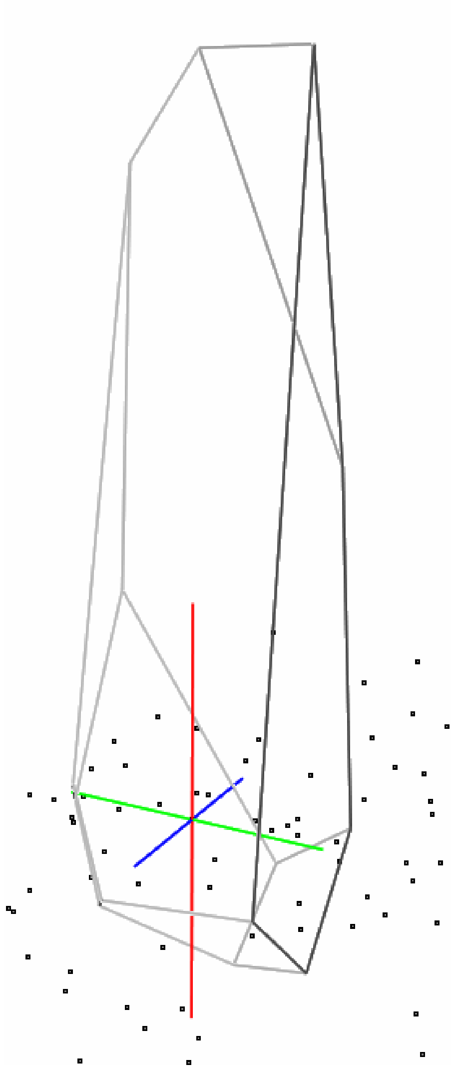
$$\int_{\Omega} (X-p)(X-p)^T dV$$

centroid

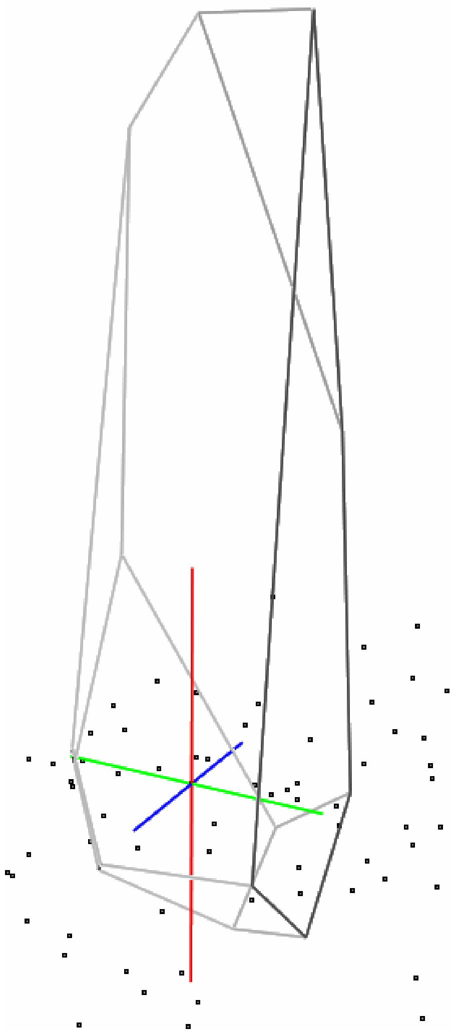
Covariance matrices of Voronoi cells

Noise-free Case



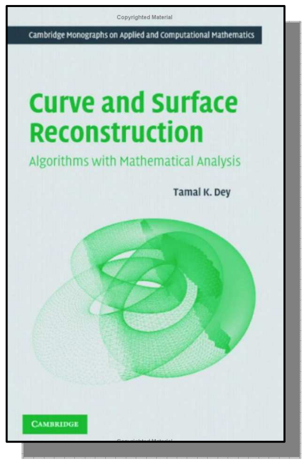


Normal Estimation: Convergence?



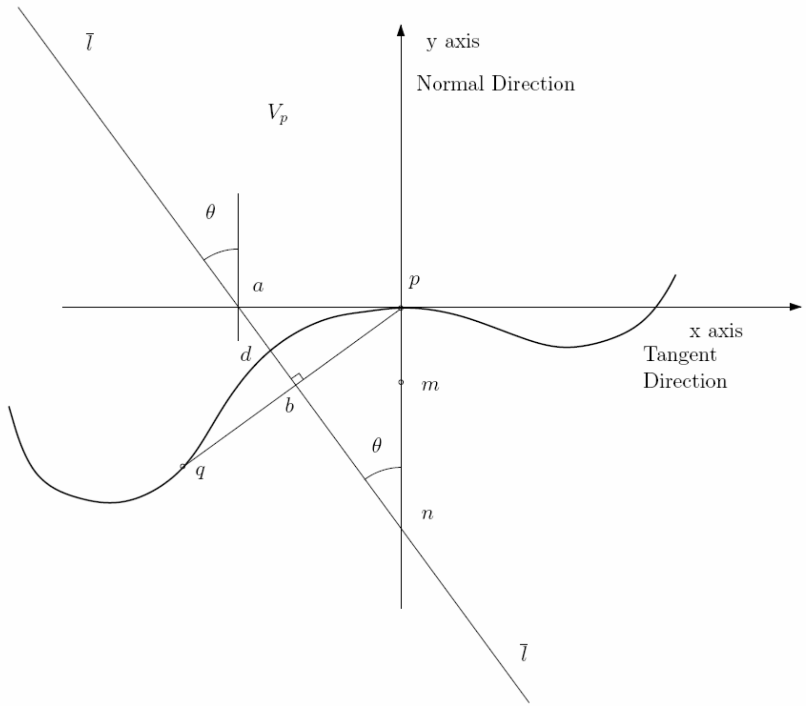
[Mitra & Nguyen]
Estimating surface normals in noisy point cloud data. SoCG ’03.

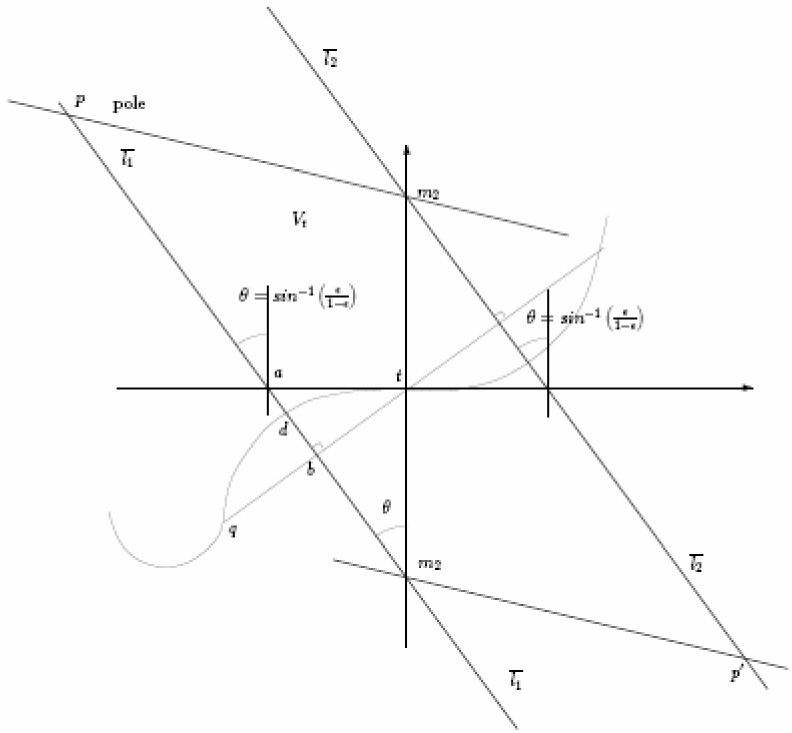
[Dey] *Curve and Surface Reconstruction : Algorithms with Mathematical Analysis*



Normal Estimation: Convergence?

- **Noise-free case** (ε-sampling): yes



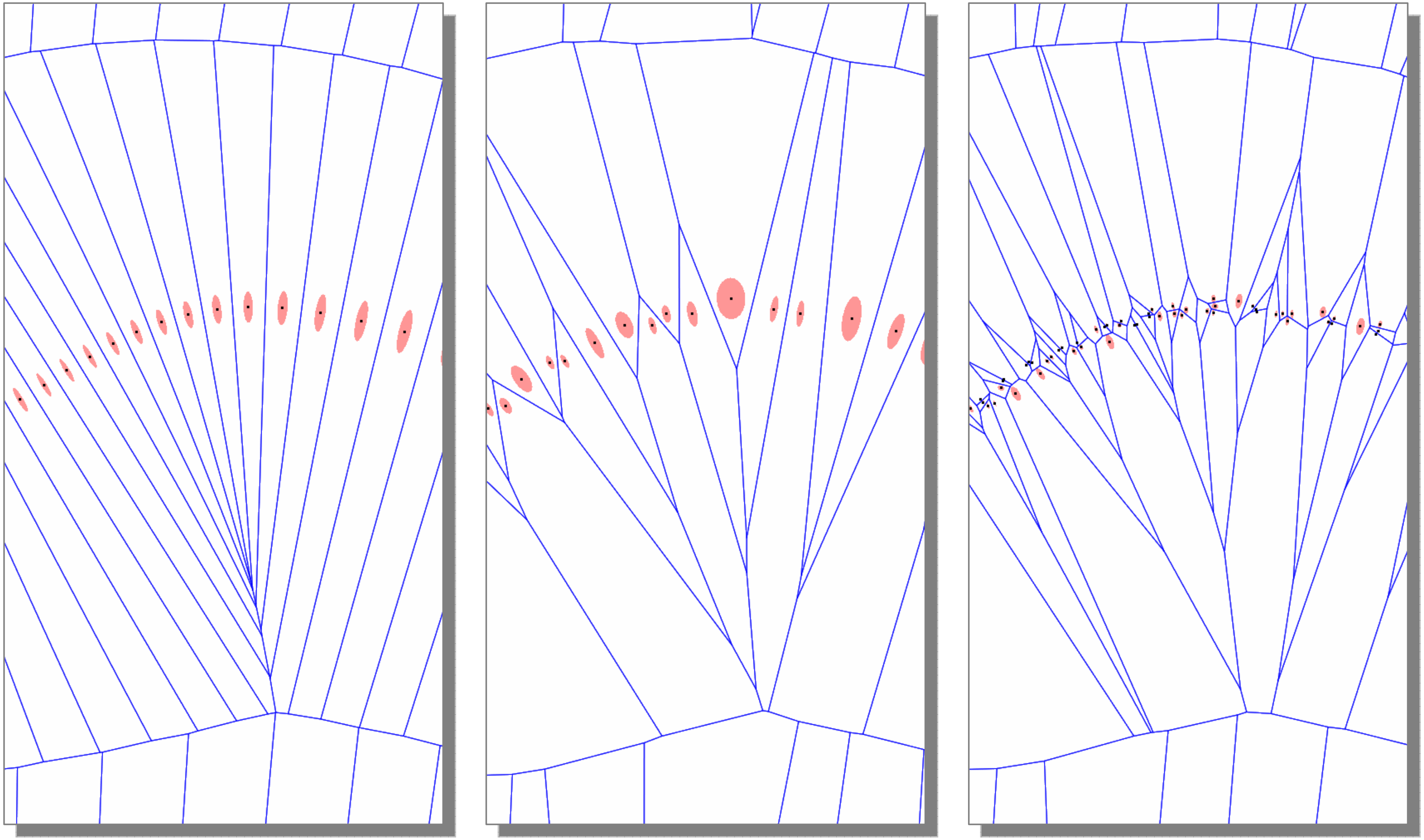


Normal Estimation: Convergence Rate?

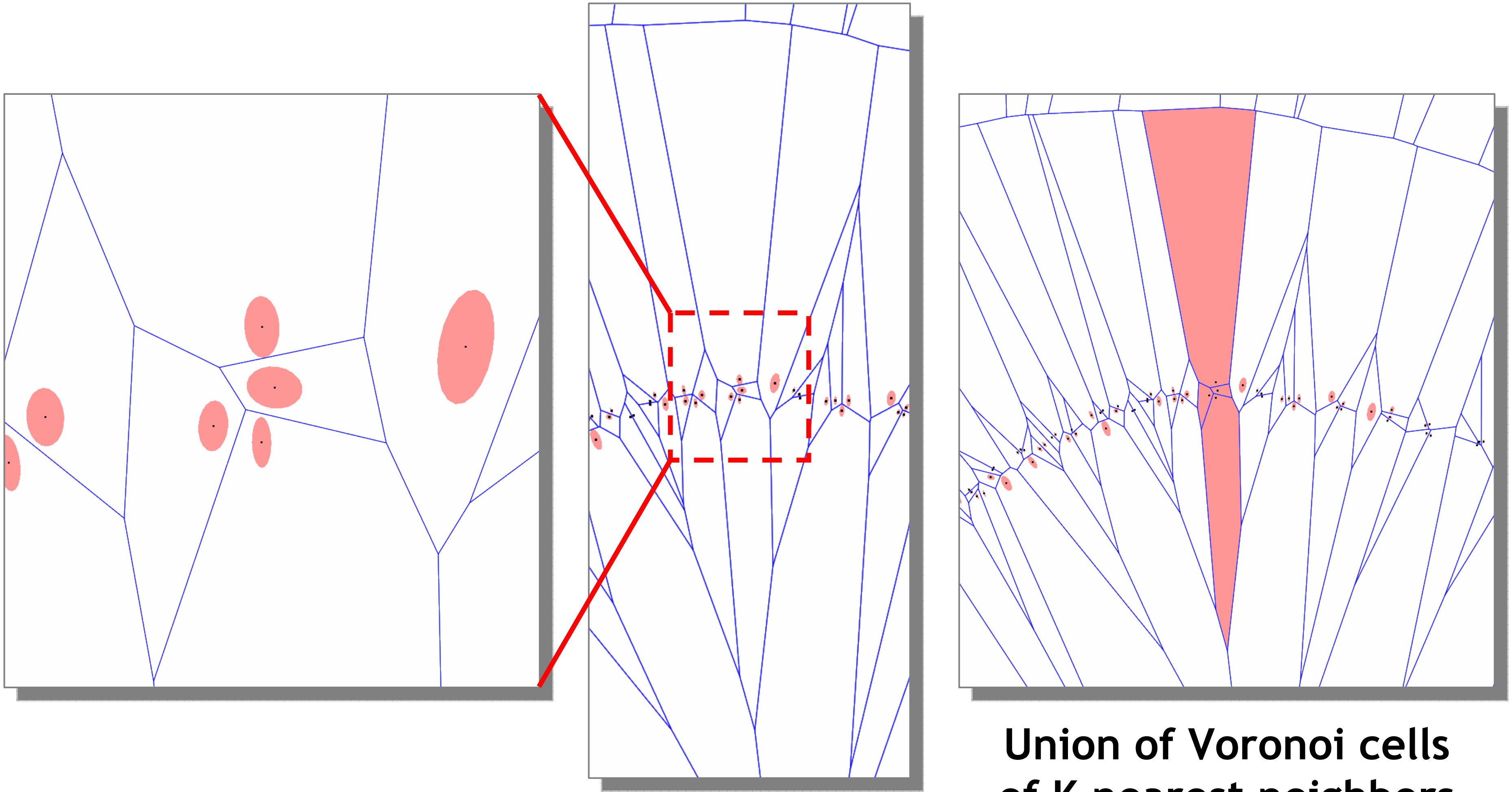
Future work:

- Better than
  - point-based PCA and variants?
  - jet fitting?
  - Others
- Noisy case?

Noise-free vs Noisy

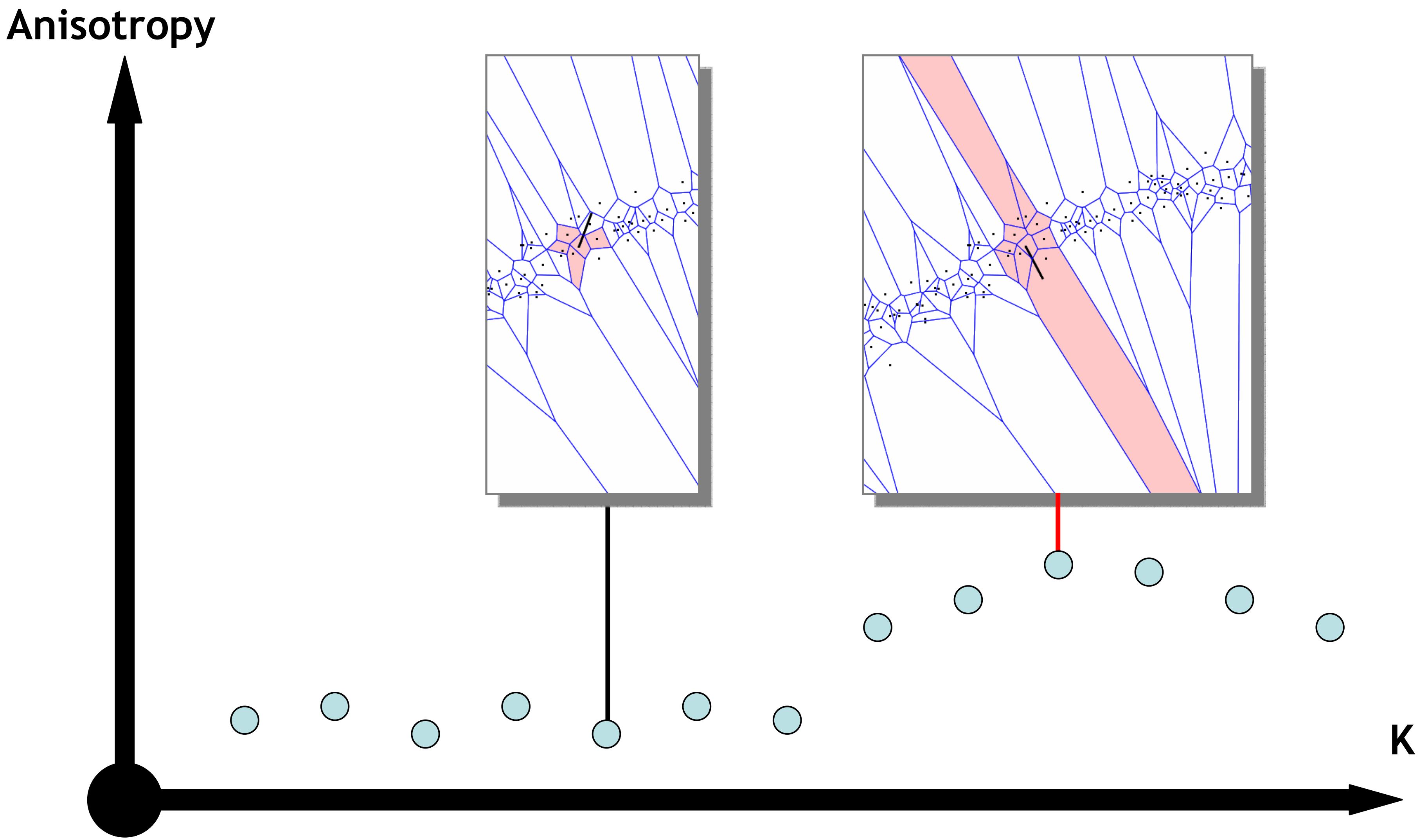


Noisy Case

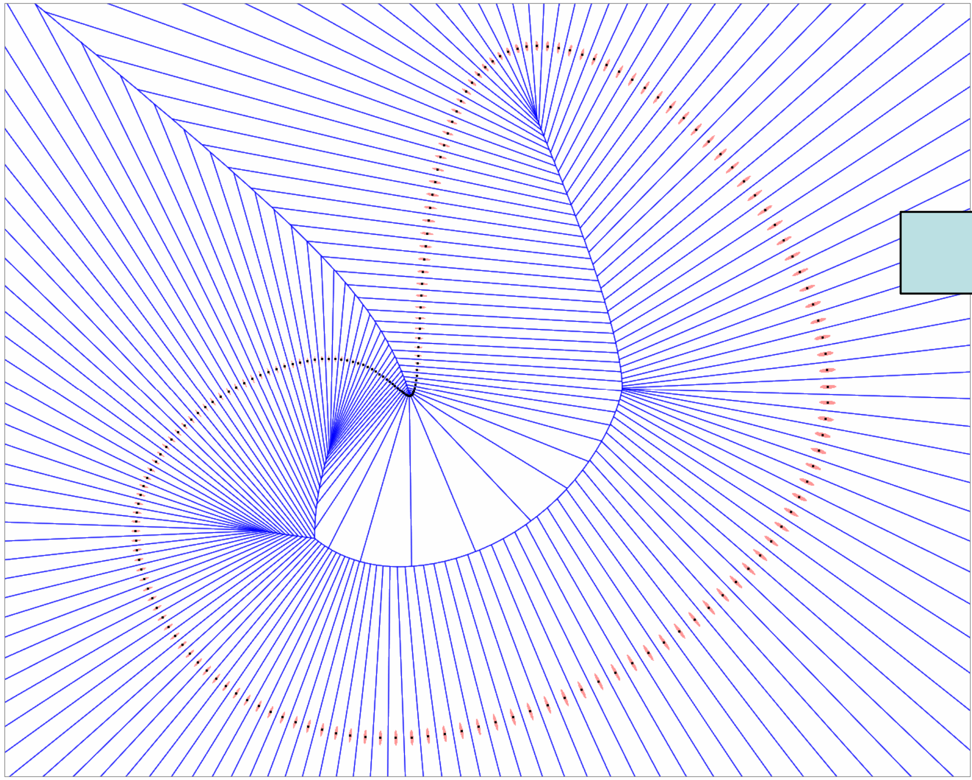


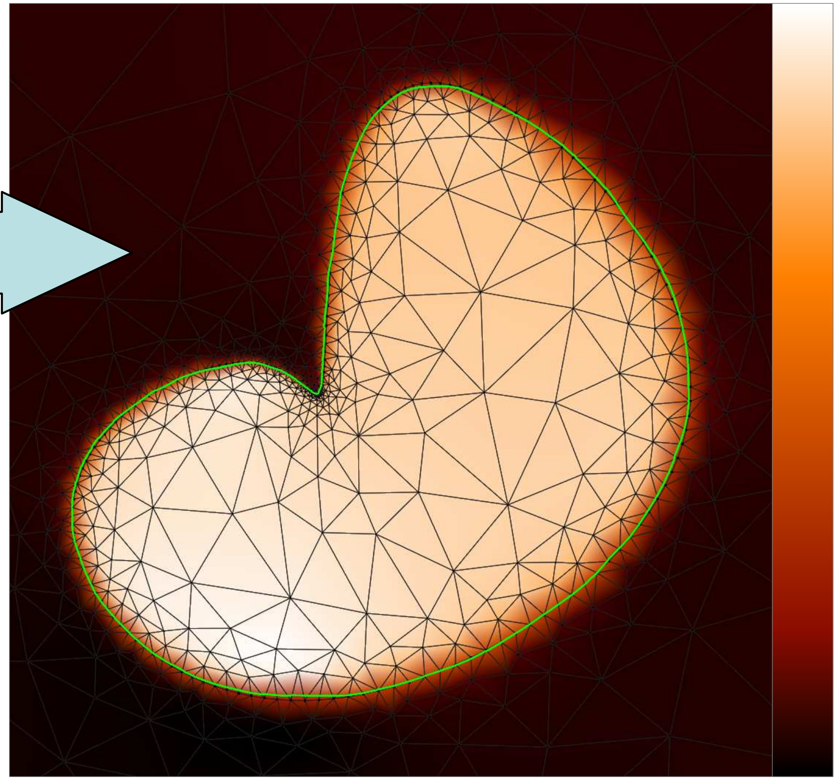
Union of Voronoi cells of K nearest neighbors

How to choose K?



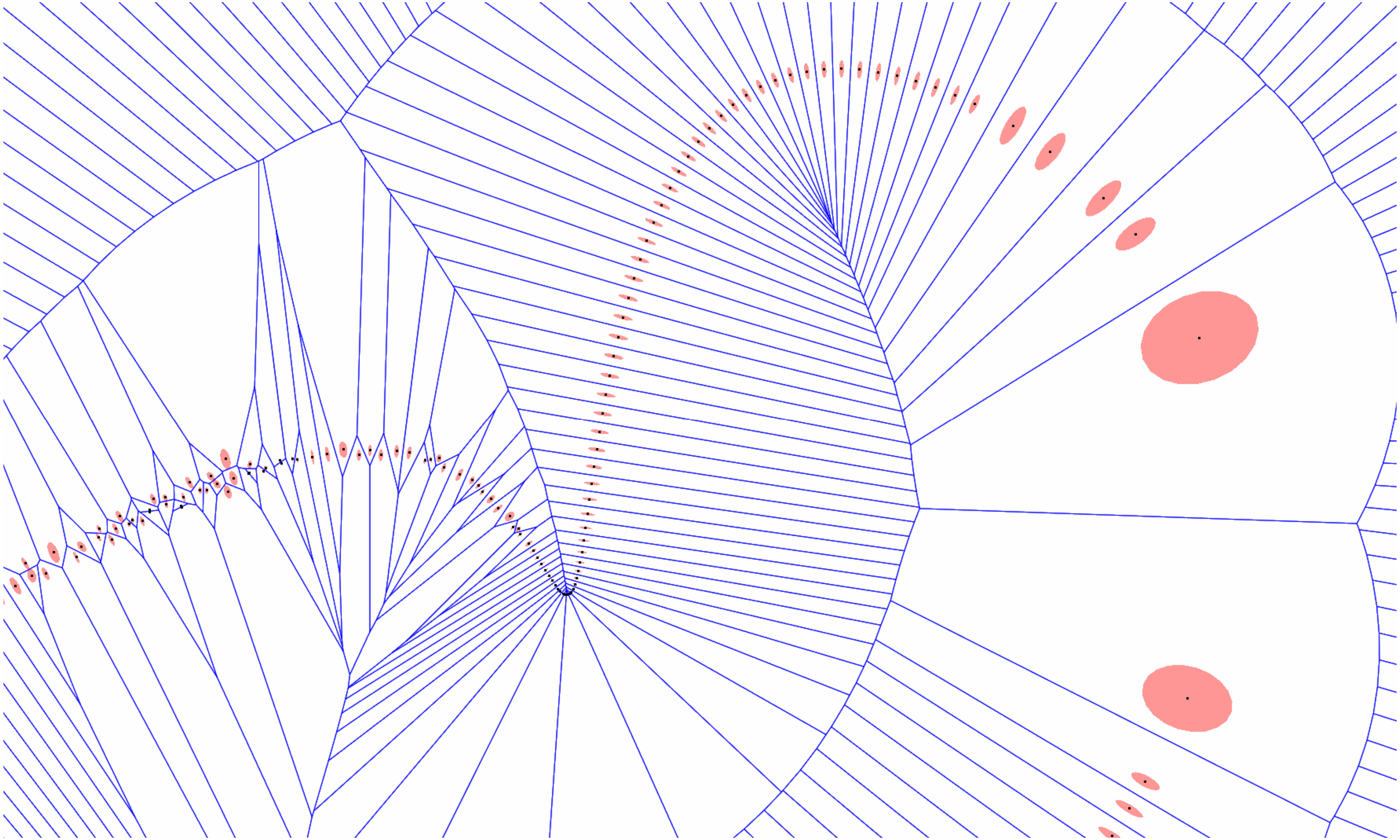
Implicit Function





Tensors **Implicit function**

Input (reminder)



Variational Formulation

- Find implicit function f such that its gradient ∇f **best aligns to the principal component** of the tensors.

Given a tensor field C, find the *maximizer* f of:

$$E_C^D(f) = \int_\Omega \nabla f^t \, C \, \nabla f \text{ subject to: } \int_\Omega \left[|\Delta f|^2 + \varepsilon |f|^2 \right] = 1$$

Biharmonic energy

Measures smoothness of ∇f

Anisotropic Dirichlet energy

Measures alignment with tensors

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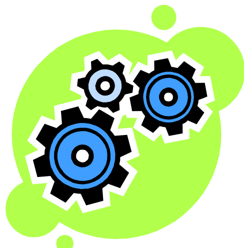
Biharmonic energy
Favors smoothness of ∇f

Anisotropic Dirichlet Energy
Rewards alignment with tensors



Regularization
Avoids constraints & improve conditioning

Rationale

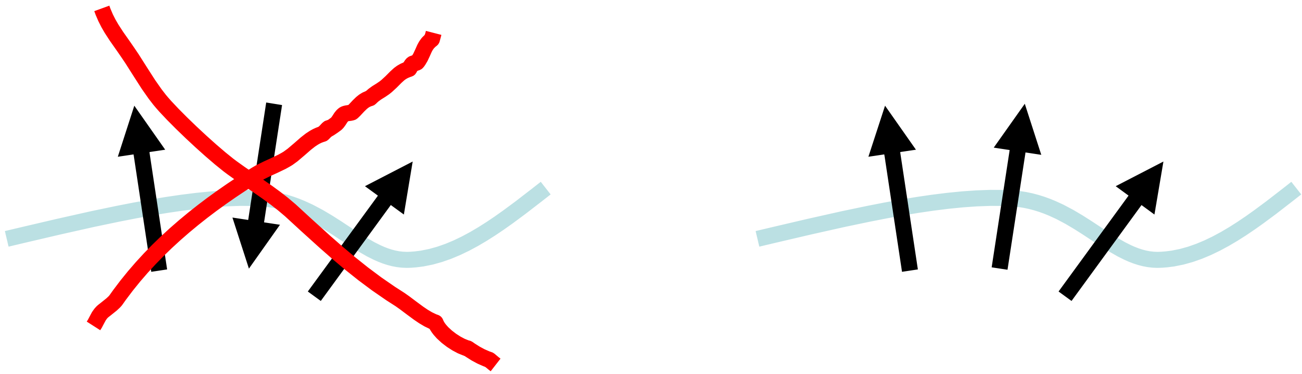
- **On areas with:**
  - **anisotropic** tensors: favor alignment
  - **isotropic** tensors: favor smoothness

Rationale

- **On areas with:**
  - **anisotropic** tensors: favor alignment
  - **isotropic** tensors: favor smoothness

Large aligned gradients + smoothness

-> consistent orientation of ∇f



Solver

Given a tensor field C, find the *maximizer* f of:

$$E_C^D(f) = \int_\Omega \nabla f^t \, C \, \nabla f \text{ subject to: } \int_\Omega \left[|\Delta f|^2 + \varepsilon |f|^2 \right] = 1$$

***A*: Anisotropic Laplacian operator**

$$E_C^D(F) \approx F^t \, A \, F$$

***B*: Isotropic Bilaplacian operator**

$$E^B(f) \approx F^t \, B \, F$$

Generalized Eigenvalue Problem

$$AF = \lambda BF$$

max

eigenvector
(PWL function)

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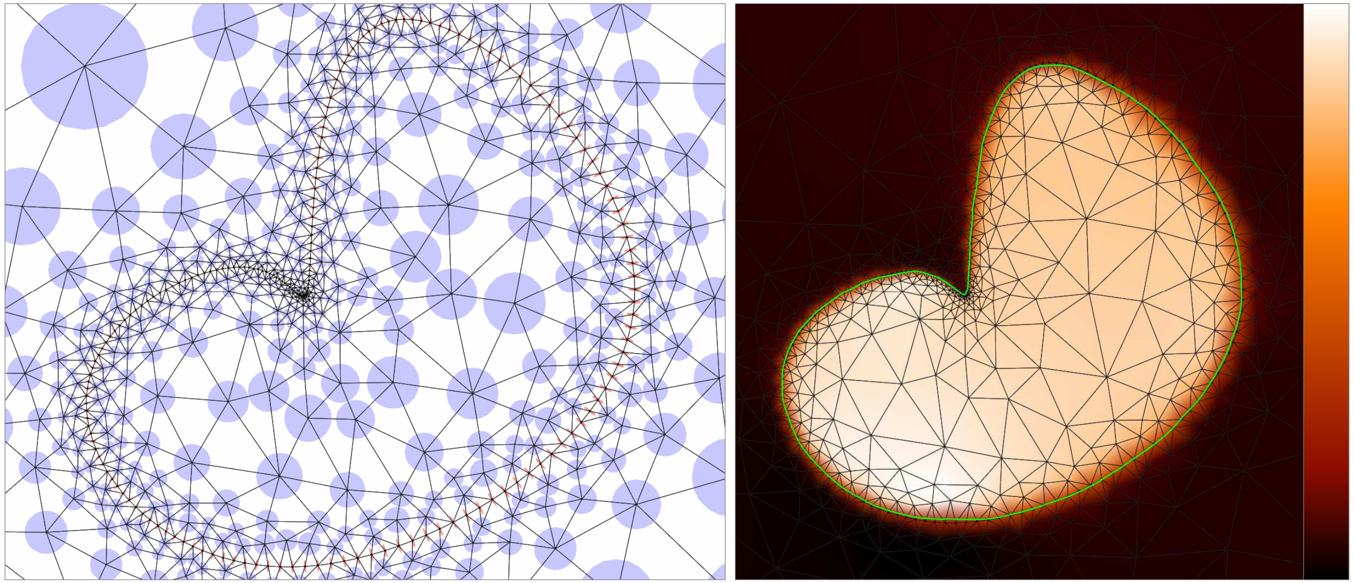
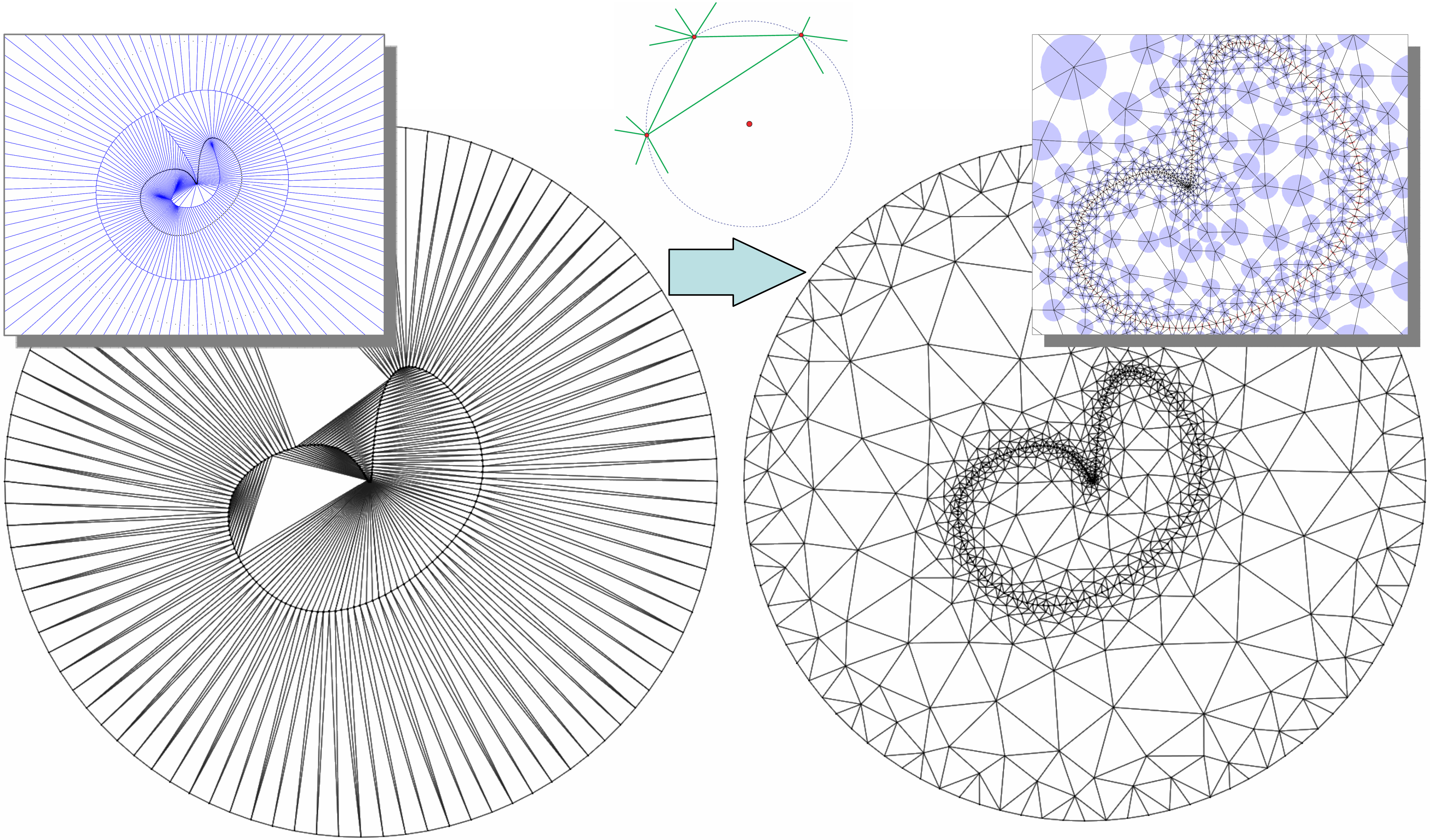
$$AF = \lambda BF$$

eigenvector
(PWL function)



Delaunay Refinement



Extensions

$$\underbrace{\text{Bilaplacian + Laplacian + data fitting}}$$

$$AF = \lambda BF$$

…Turned into Std Eigenvalue Problem

Compute Cholesky factorization of *B* **[TAUCS]**

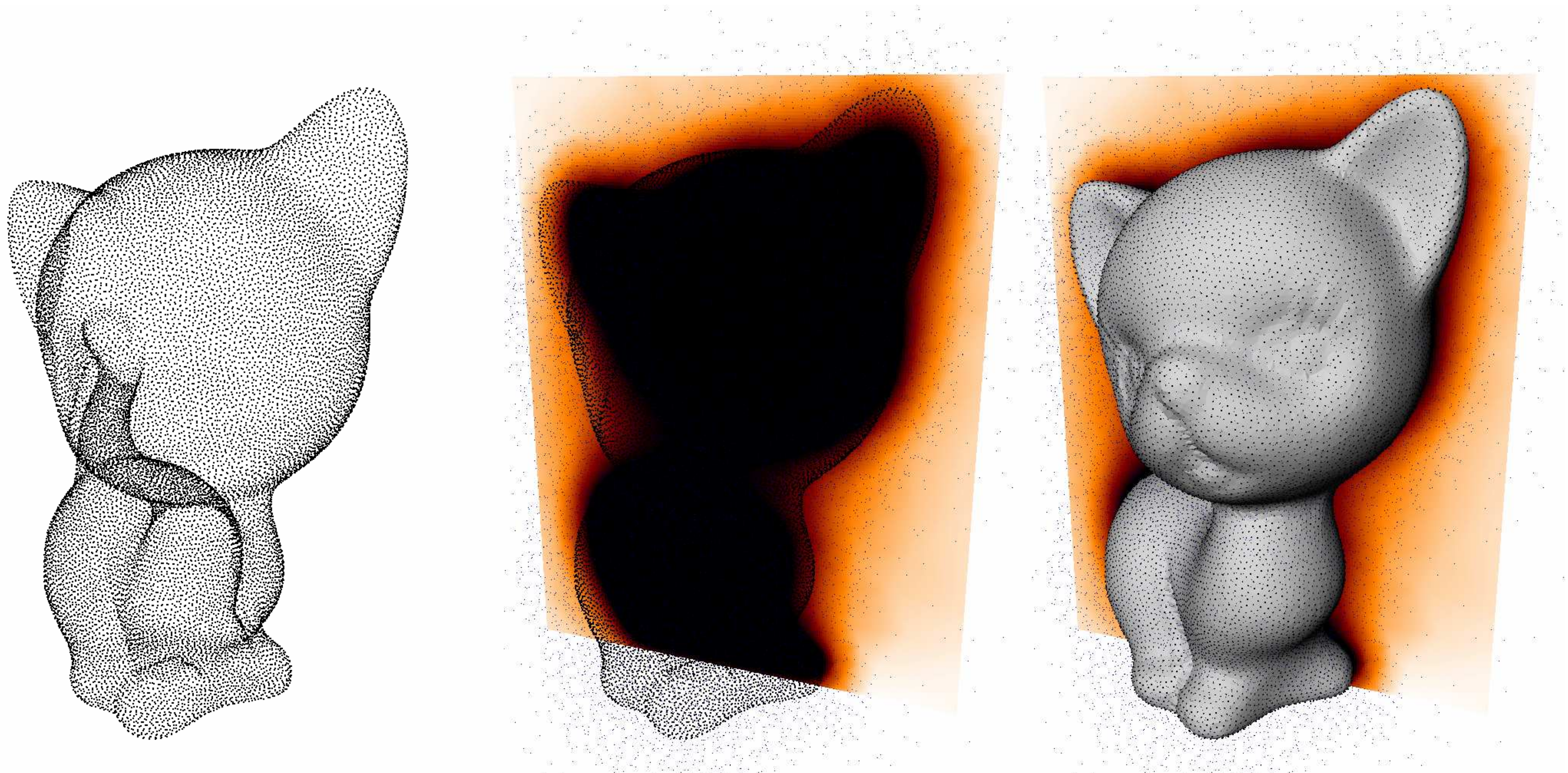
$$B = LL^t$$

$$AF = \lambda LL^t F \Leftrightarrow L^{-1} A L^{-t} L^t F = \lambda L^t F \Leftrightarrow \begin{cases} L^{-1} A L^{-t} G = \lambda G \\ G = L^t F \end{cases}$$

Solver:

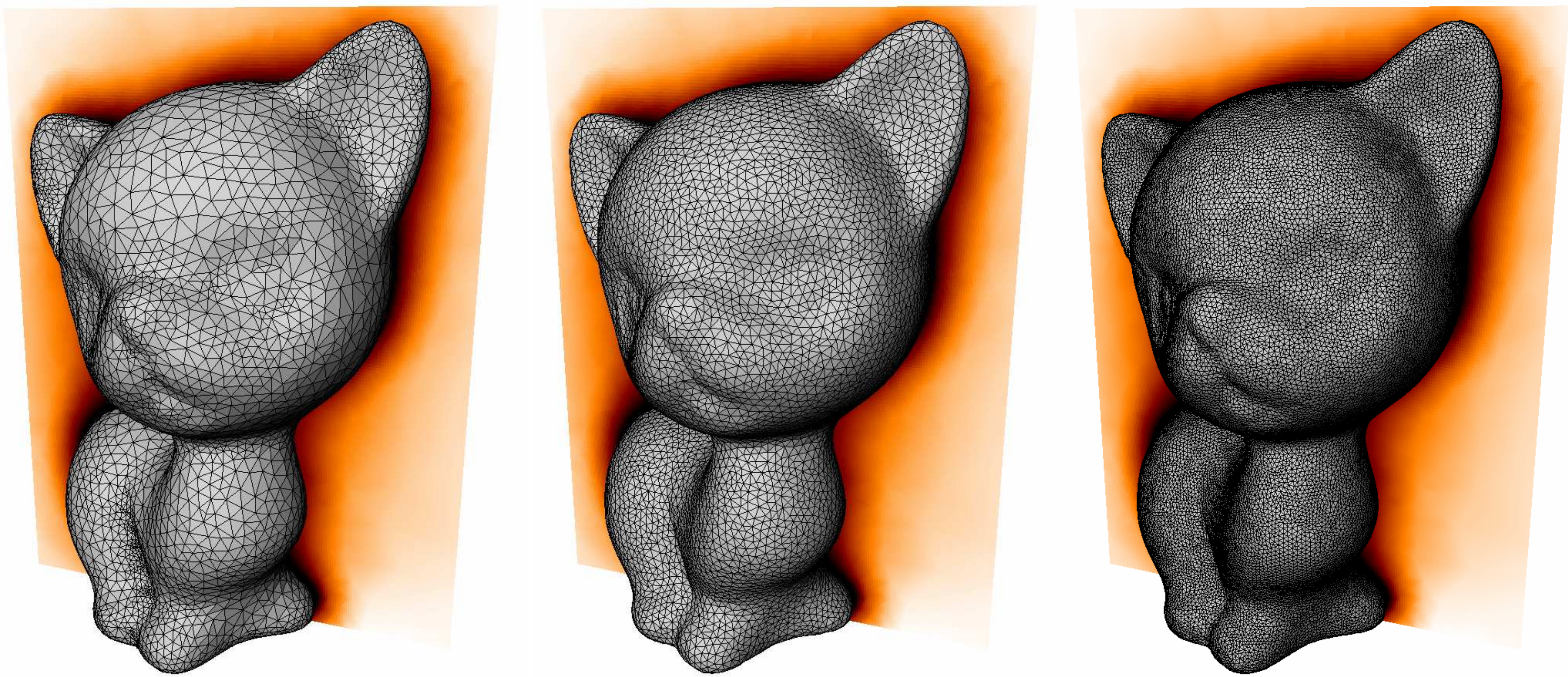
- Implicitly restarted Arnoldi method **[ARPACK++]**

Contouring



Implicit function Marching tets

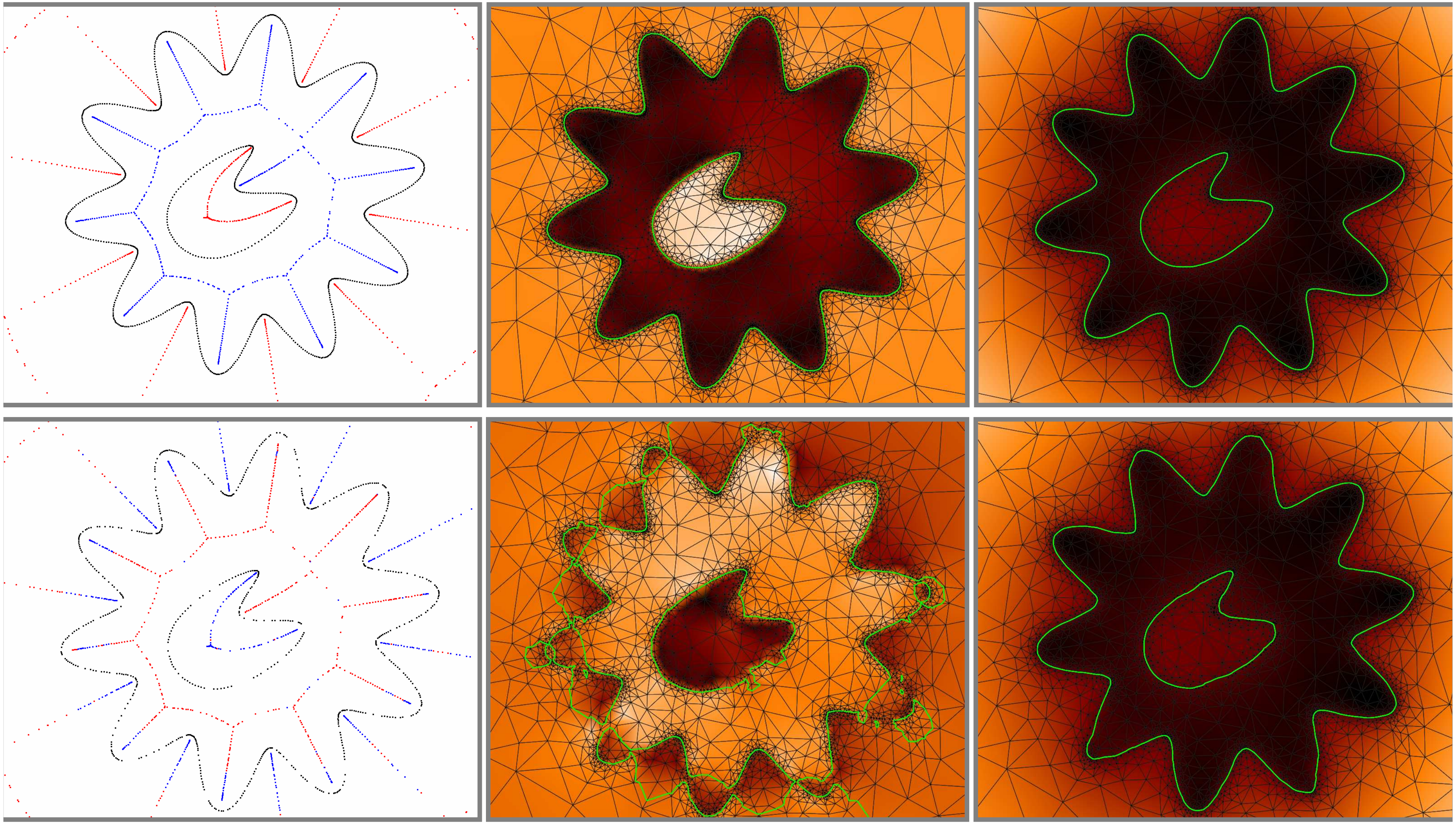
Surface Mesh Generation



Delaunay-based surface mesh generator [Boissonnat-Oudot, CGAL]

Experiments

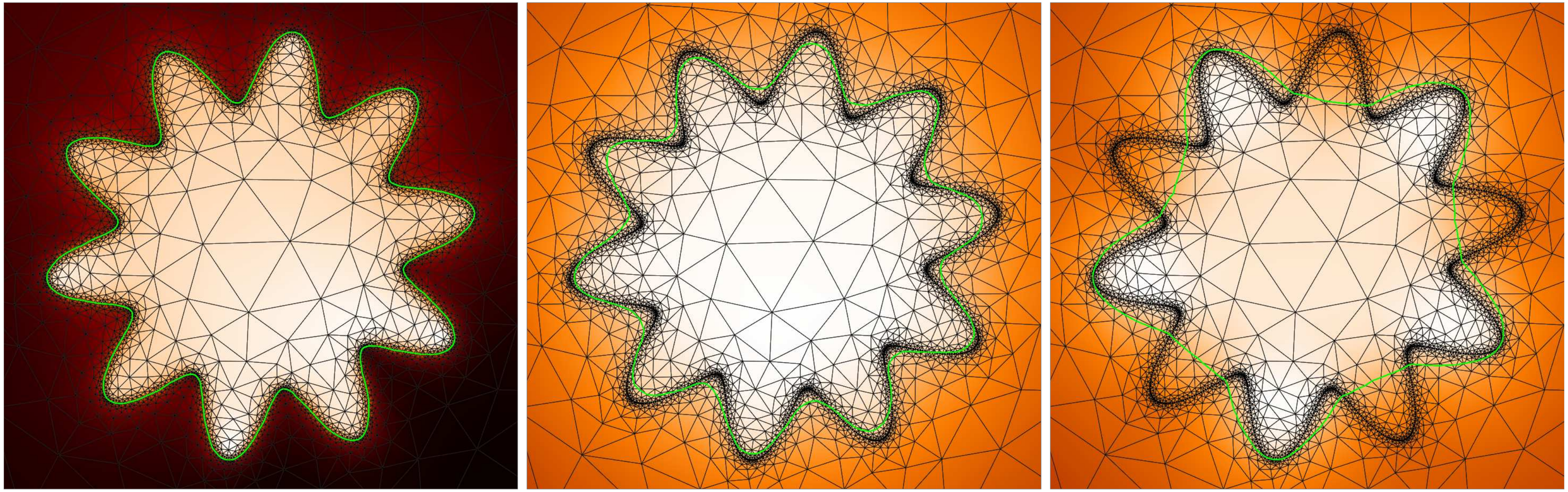
vs Poisson Reconstruction



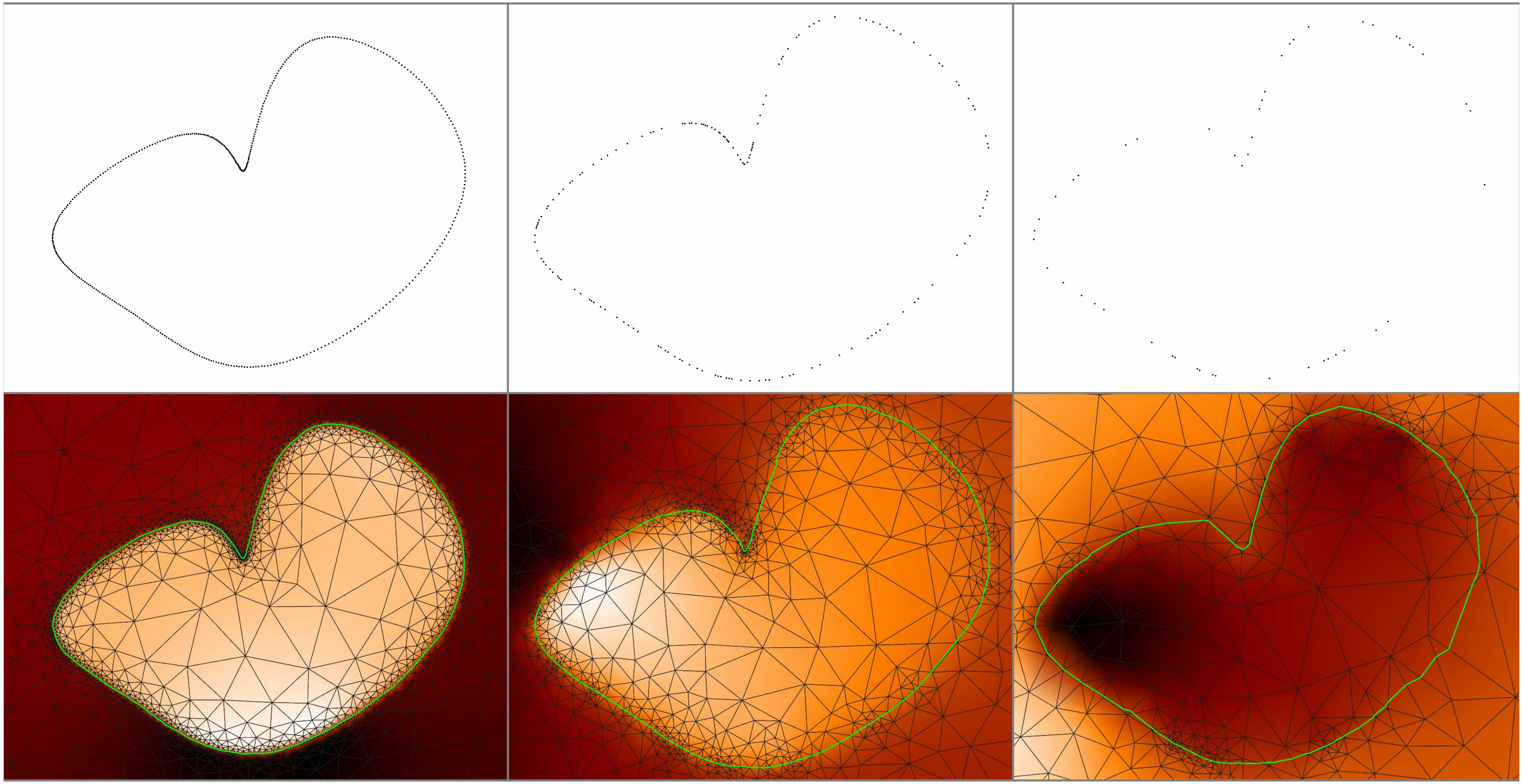
Poisson reconstruction (on simplicial mesh)

GEP

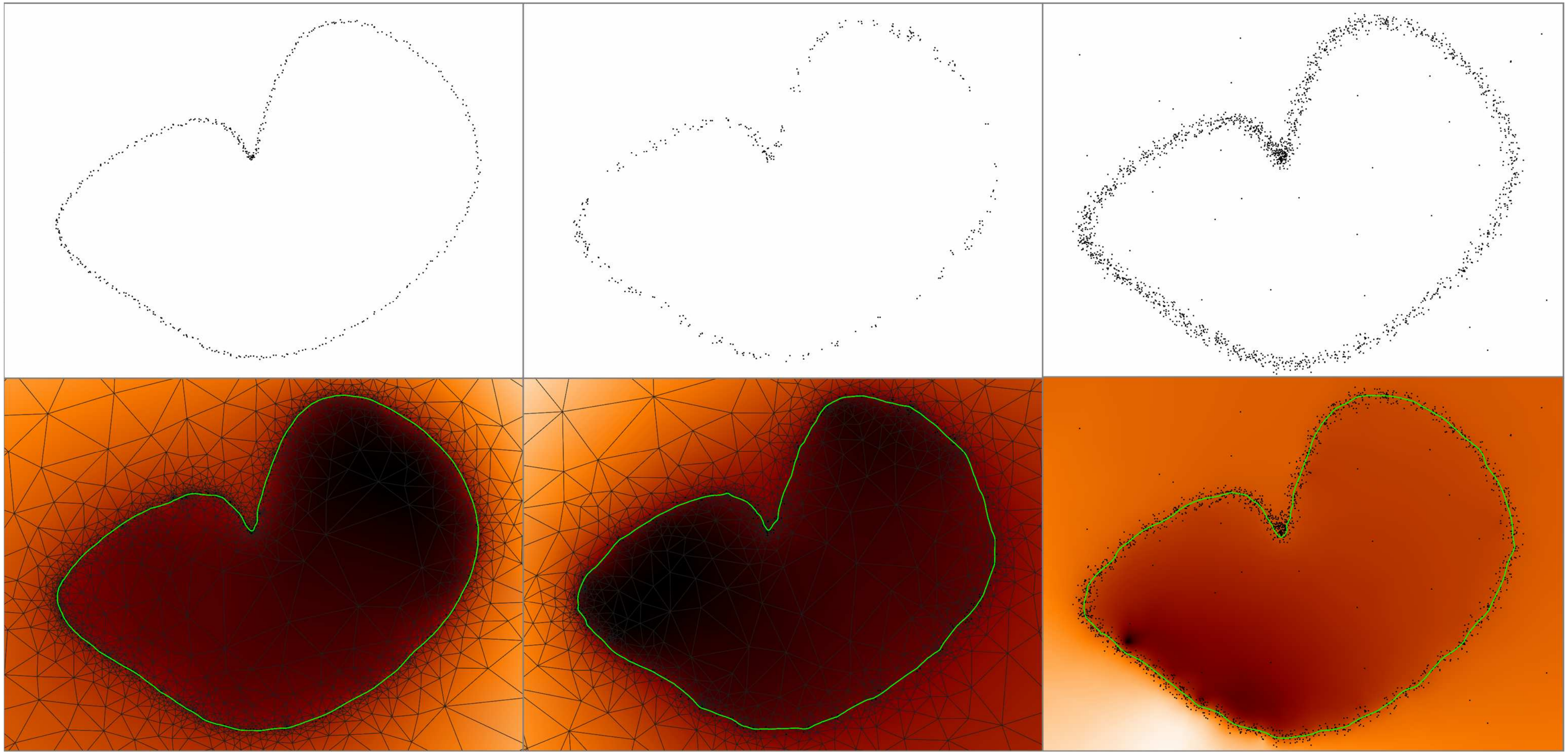
Increasing Bilaplacian



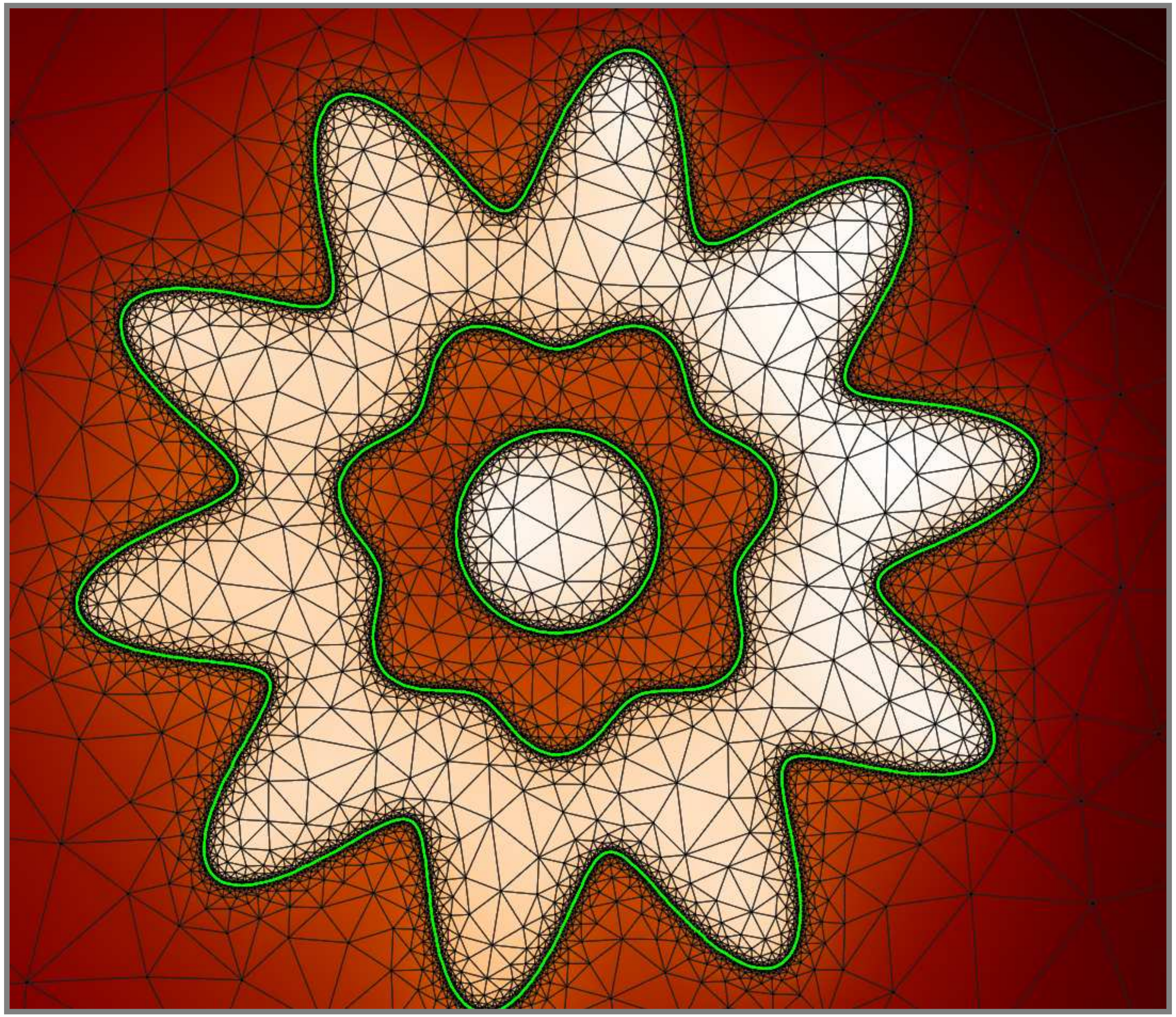
Sparse Sampling



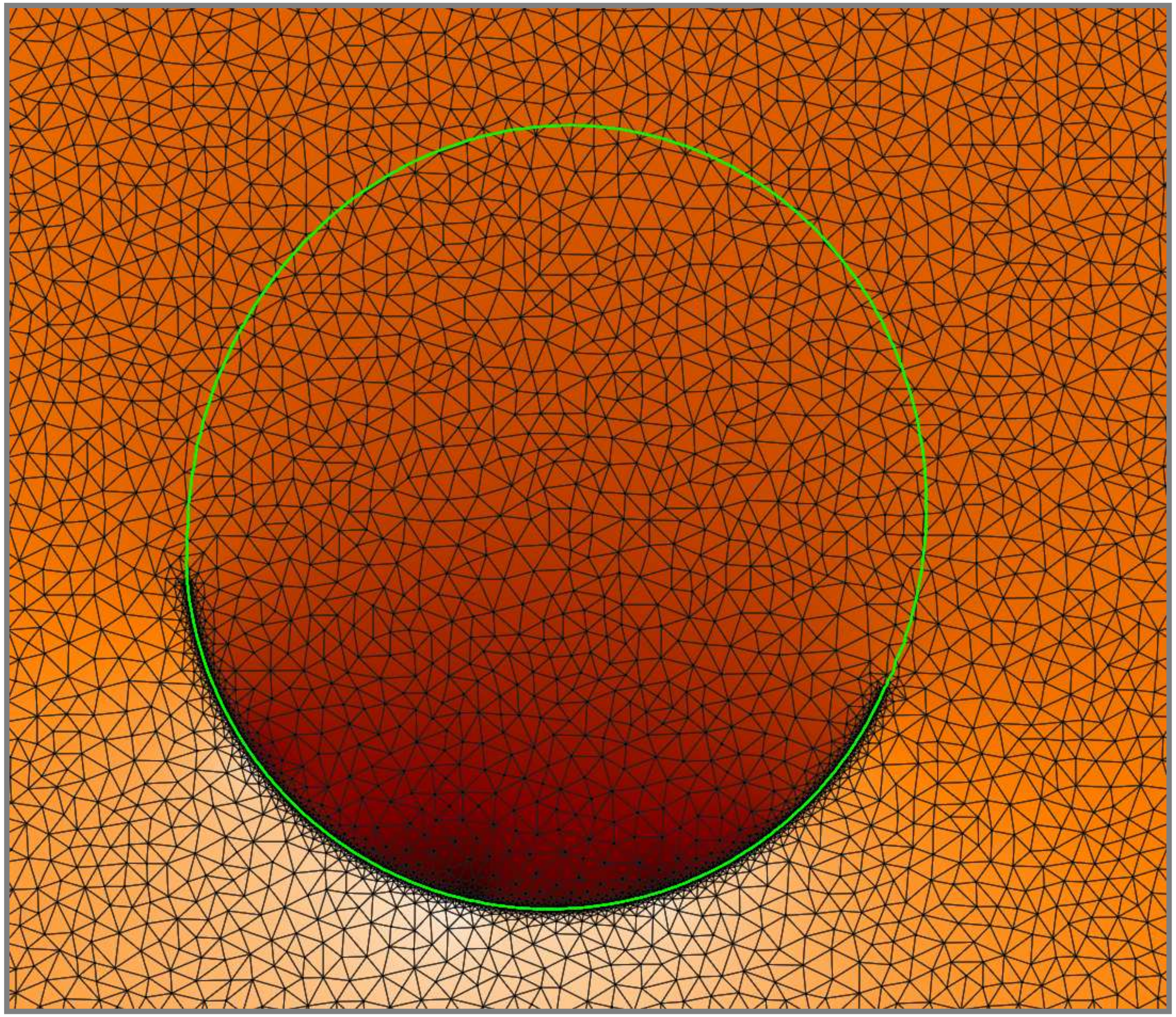
Noise



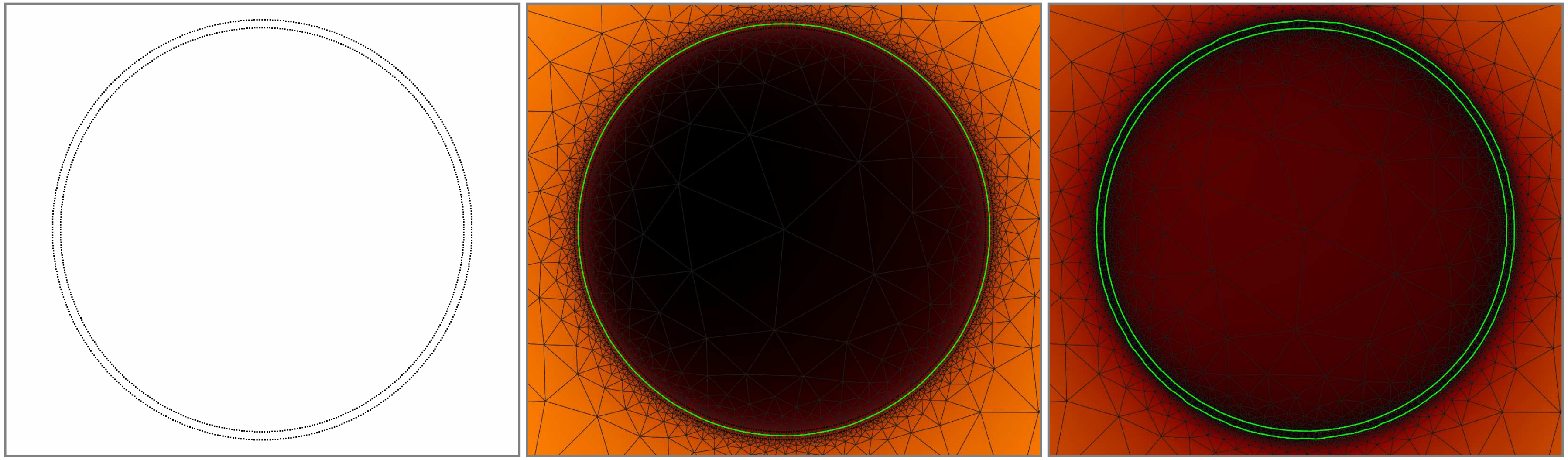
Nested Components



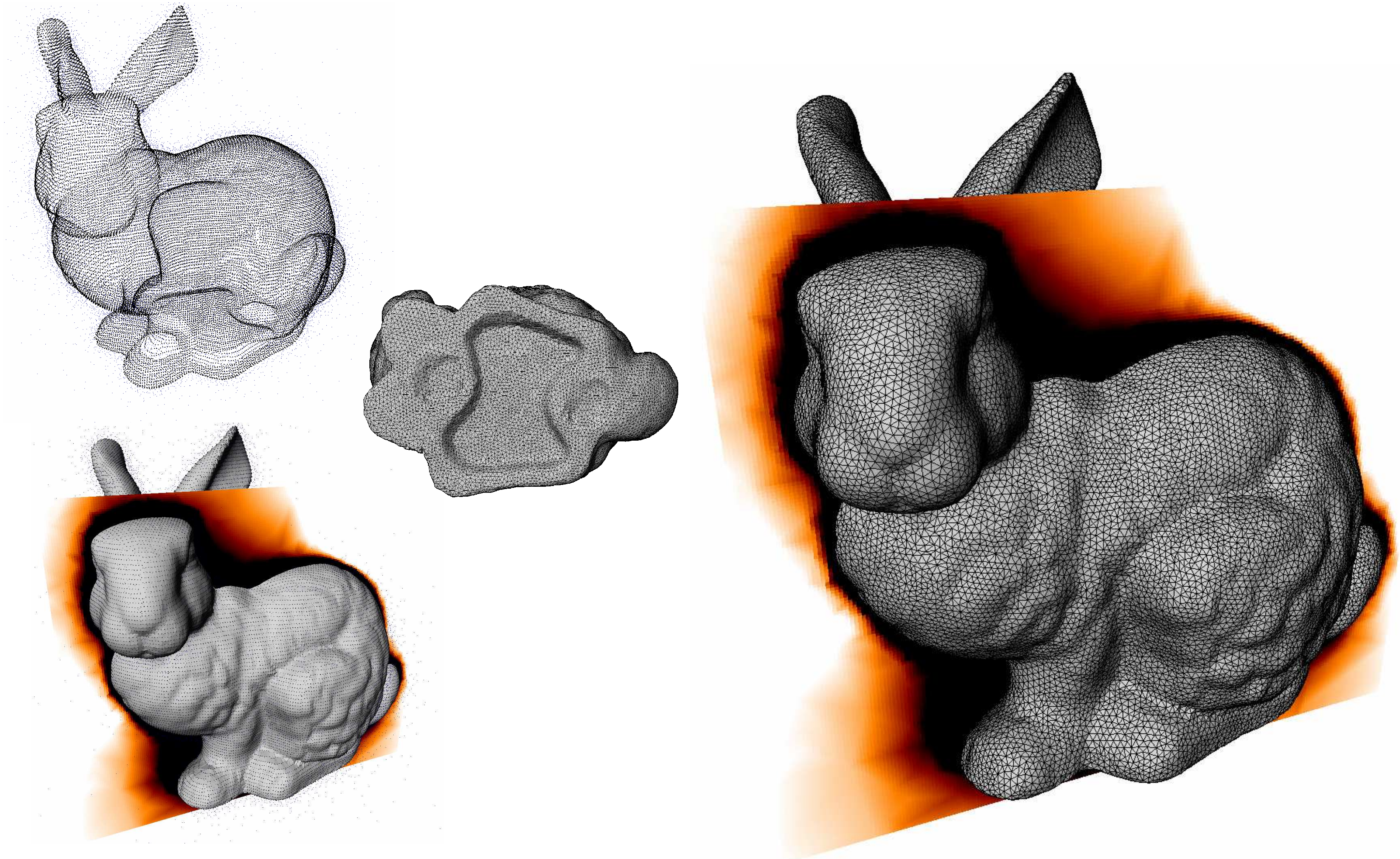
Spline-under-tension Effect



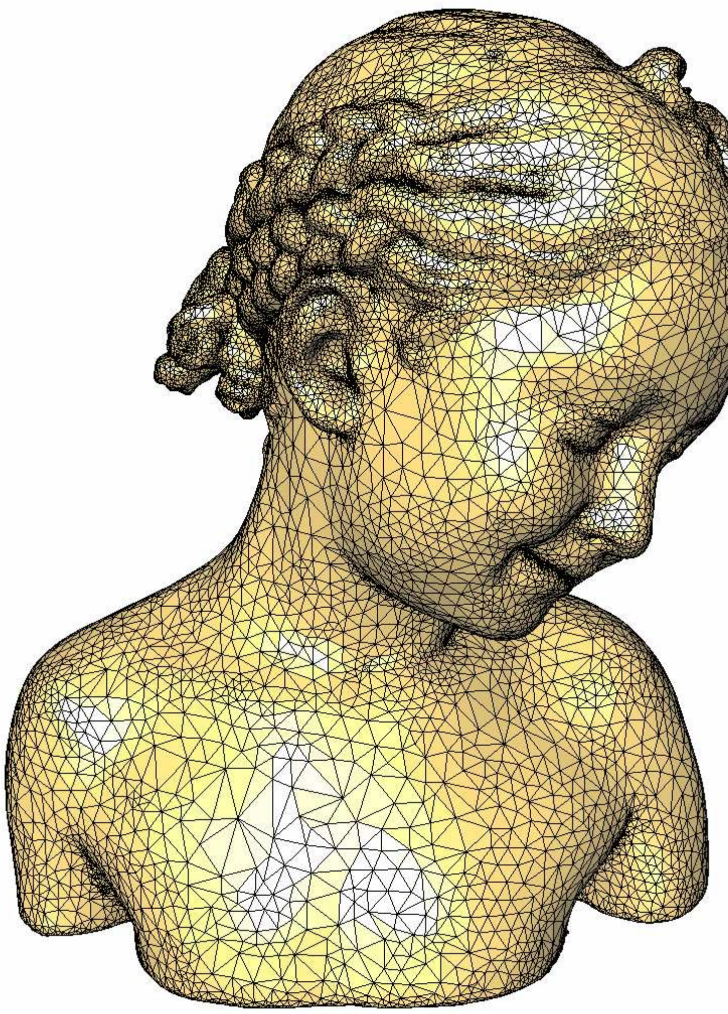
Adjustable Data Fitting

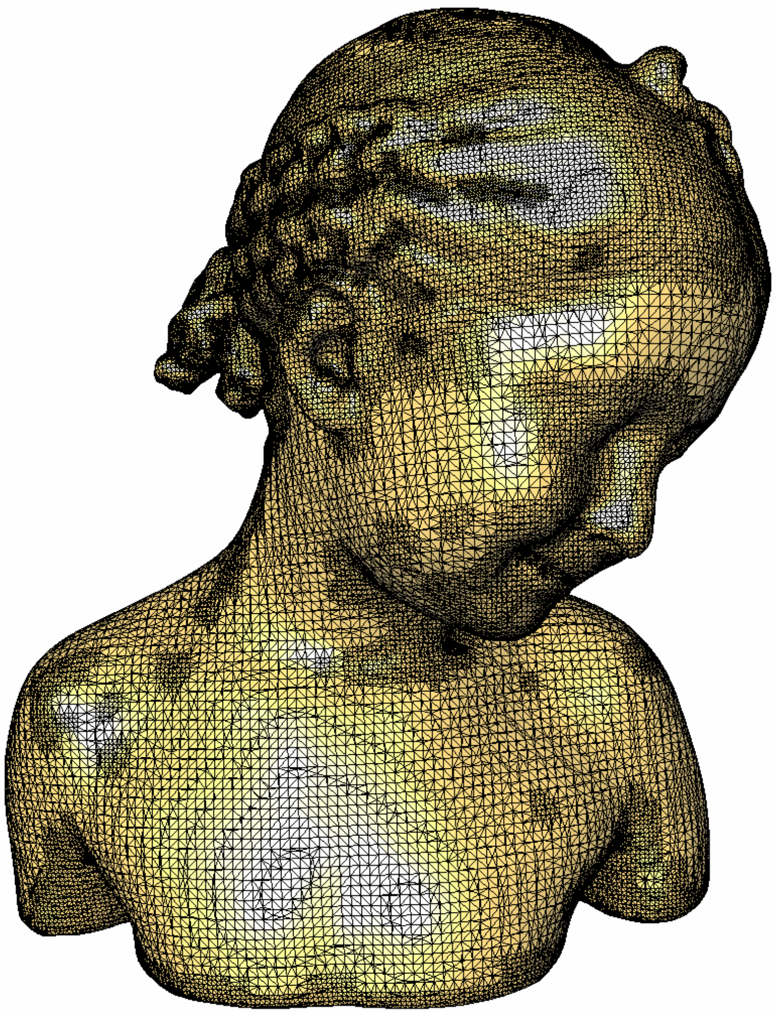


Stanford Bunny



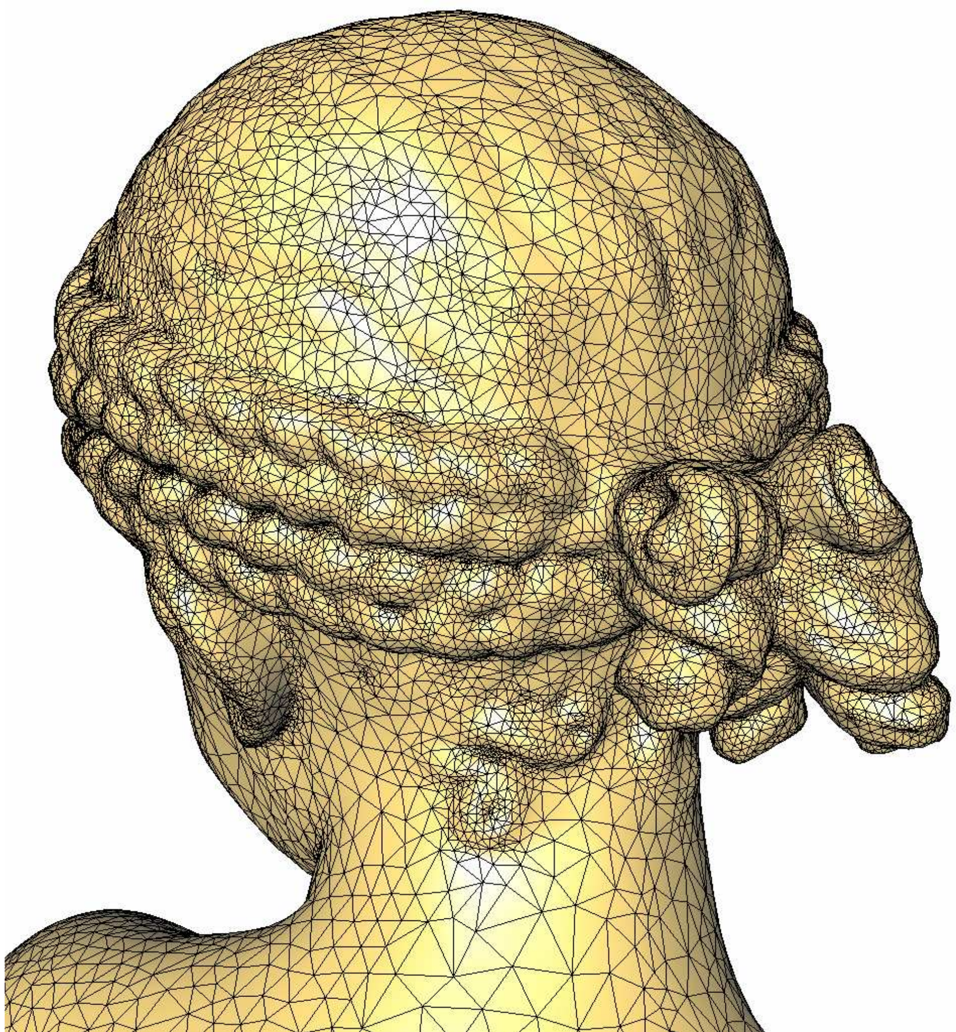
Bimba con Nastrino

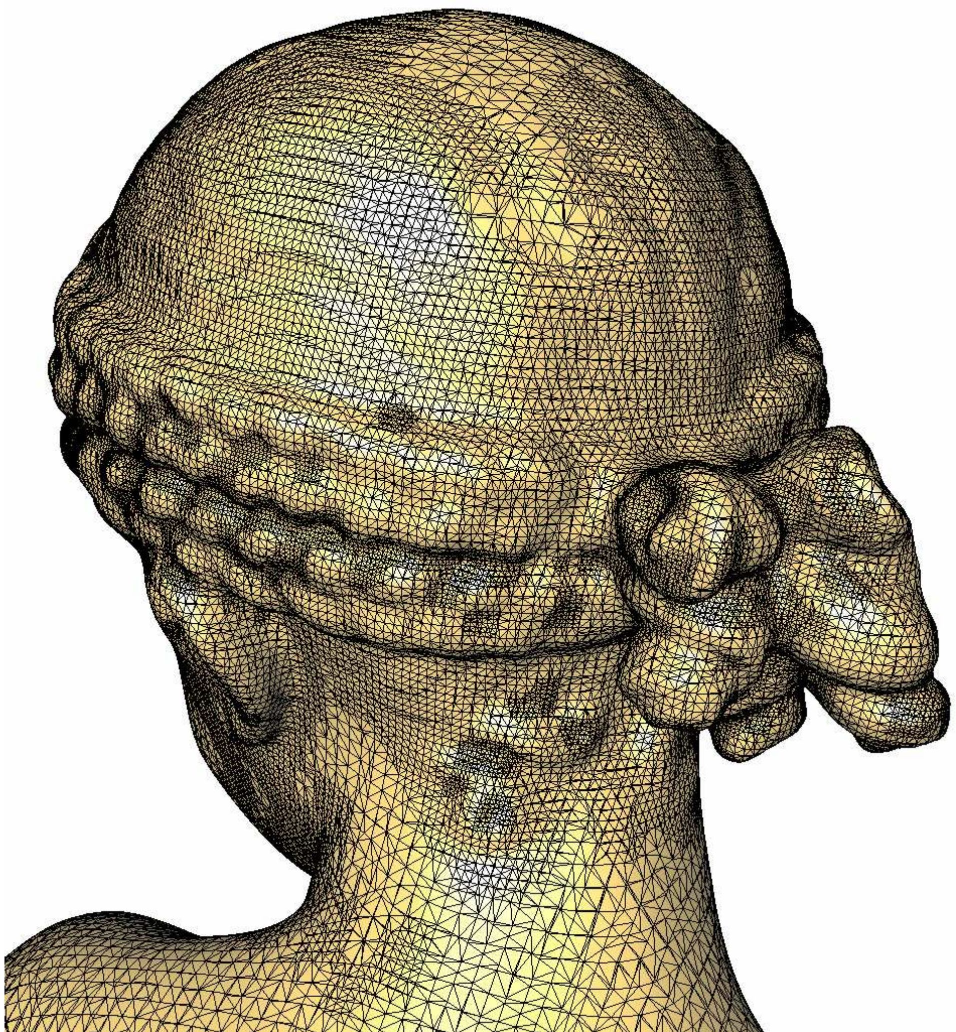




Poisson Reconstruction

Bimba con Nastrino





Bimba con Nastrino

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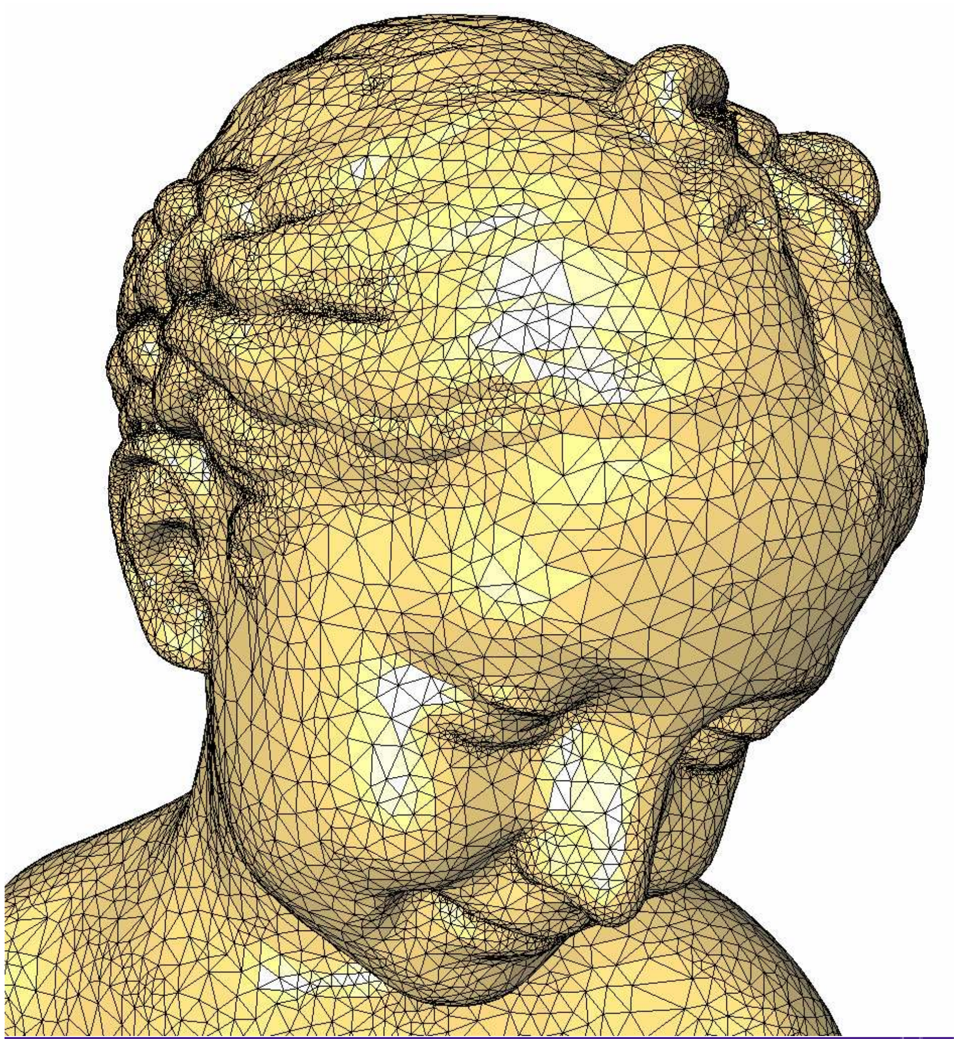

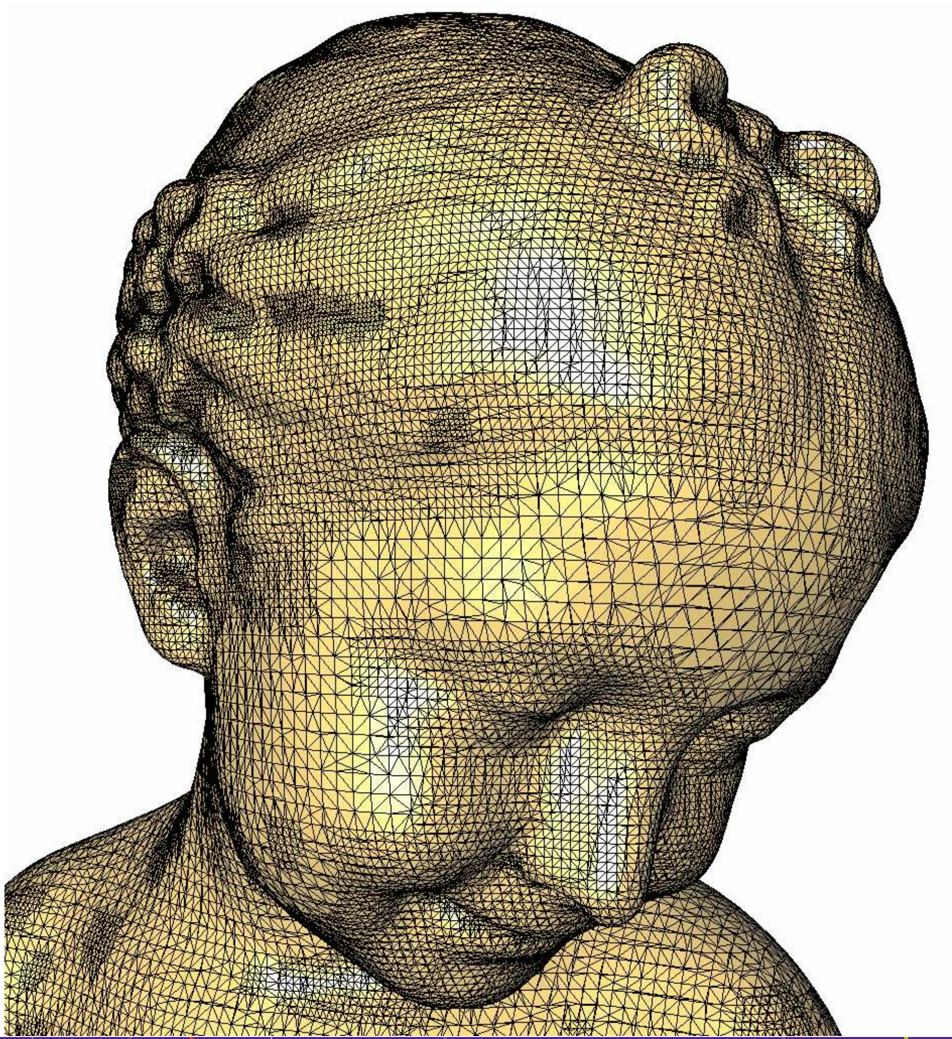

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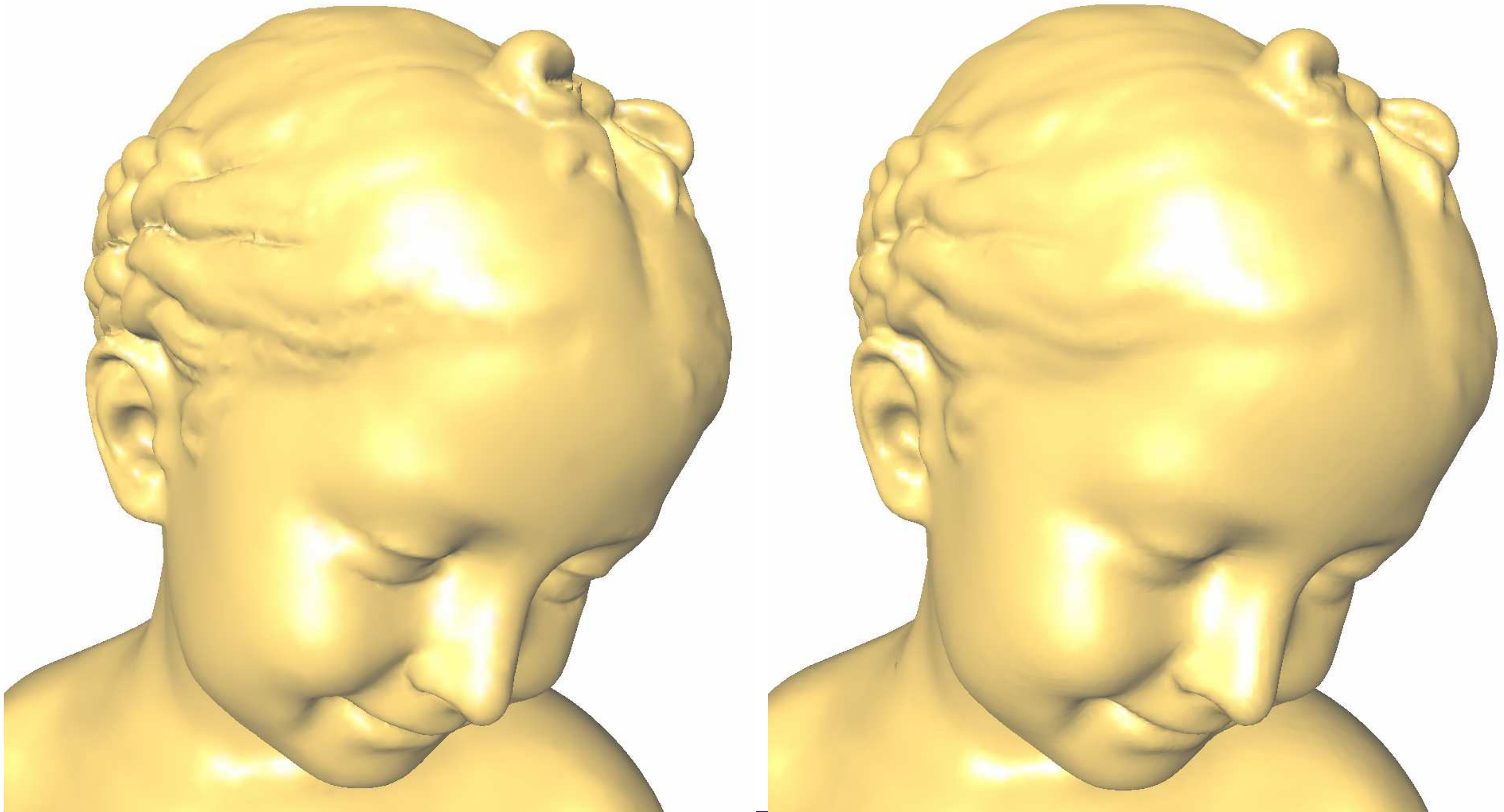
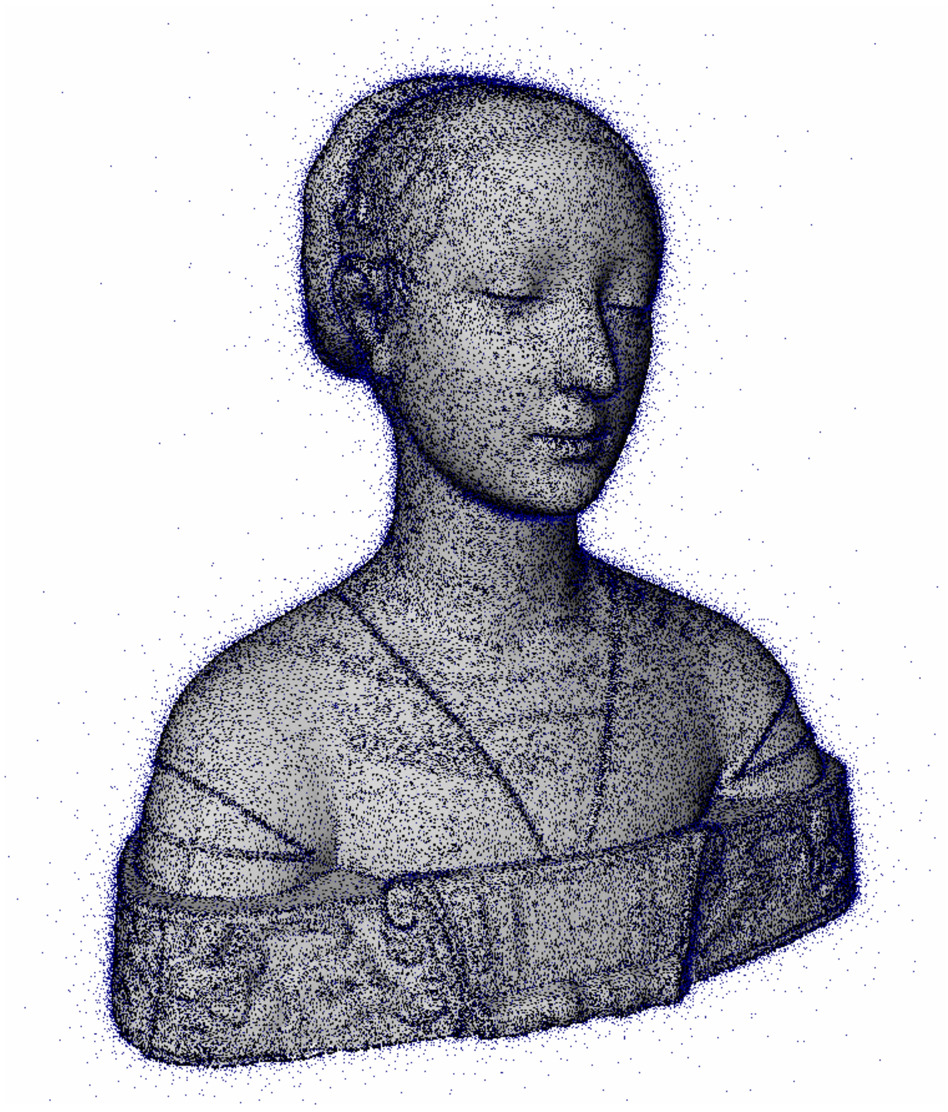
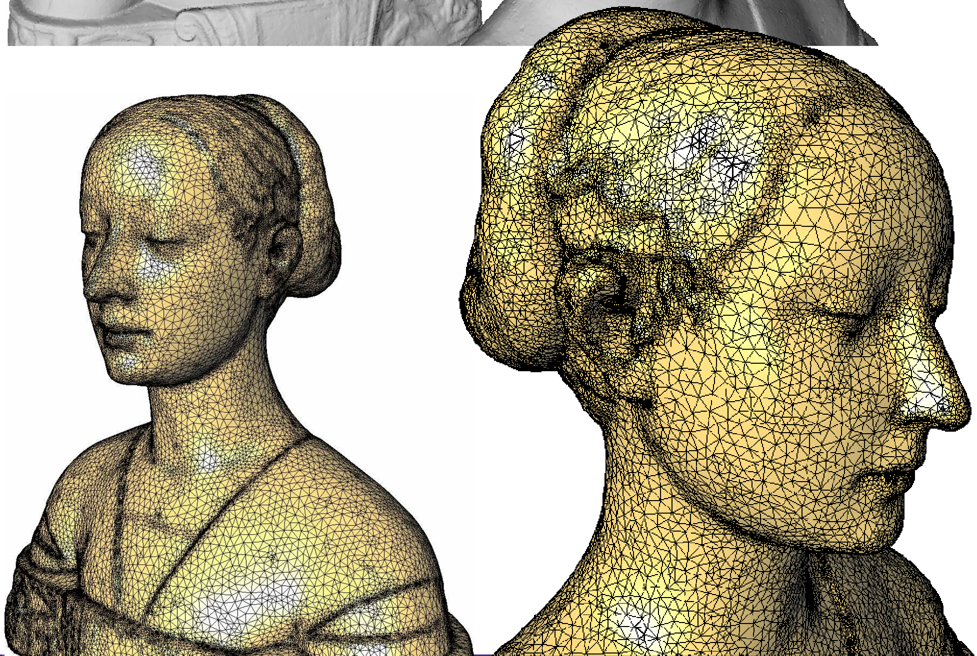


Sforza (250K points)





Conclusion

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- **Pros**
  - Handles unoriented point sets
  - Approximating
  - Adjustable smoothness vs fitting

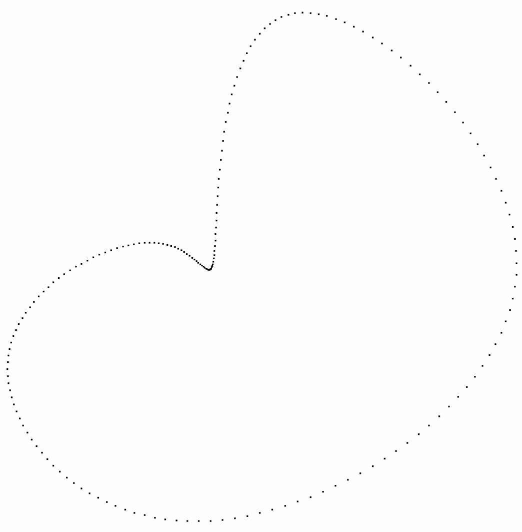
- **Cons**
  - Slow (50x Poisson reconstruction)
  - Scalability issues
    - bottleneck Cholesky factorization (max 250K points / 32bit)
    - In-core matrix reordering (METIS)

Future Work

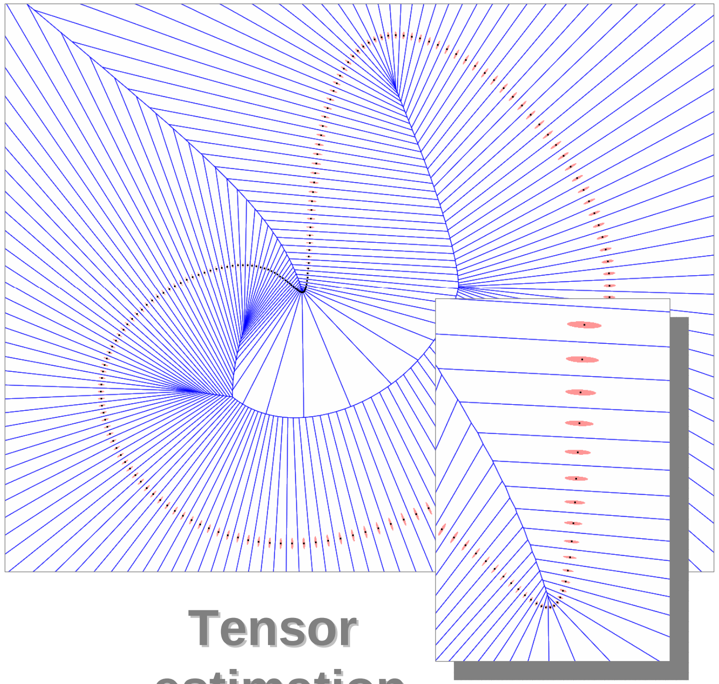
- Analysis of Voronoi-PCA normal estimation
- Improve solver
- CGAL component

Acknowledgements

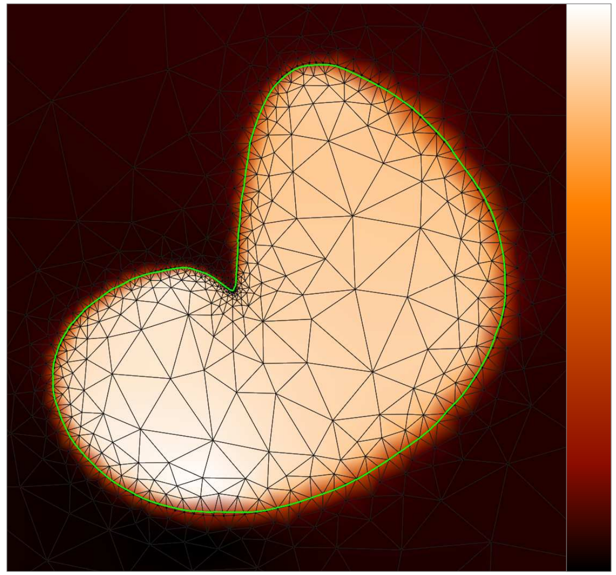
- David Bommes (RWTH Aachen)
- Mario Botsch (ETH Zurich)



Point set



Tensor estimation



Implicit function + contouring