On Soft Foundations for Geometric Computation

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For over two decades, Exact Geometric Computation (EGC) has provided a paradigm in Computational Geometry for the correct implementation of geometric algorithms. It is the most successful approach for addressing numerical nonrobustness, leading to successful libraries and practical algorithms in many areas. We review some pressing reasons to extend this paradigm:

* EGC algorithms may not be Turing computable (e.g., transcendental functions)
* EGC may be too inefficient (e.g., shortest path problems)
* EGC entails numerous/difficult algebraic analysis (e.g., Voronoi diagram of polyhedra)
* Exact computation is unnecessary/inappropriate (e.g., robot motion planning)

This talk describes a research program to develop “soft” (i.e., purely numerical) approaches for addressing the above issues. We illustrate these ideas by looking at recent work in several areas:

* root isolation and clustering (ISSAC’09,’11,’12,’16, SNC’11, CiE’13)
* isotopic approximation of curves and surfaces (SoCG’09,’12, SPM’12, ICMS’14)
* Voronoi diagrams (ISVD’13, SGP’16)
* robot motion planning (SoCG’13, WAFR’14, FAW’15)

Some common themes emerge from the above list: we replace the exact algebraic model of computation (Real RAM or BSS Model) by a model based on numerical interval approximations. The main algorithmic paradigms are subdivision and iteration. We introduce an input resolution parameter (epsilon), but it must be used in a novel “soft way”. Soft versions of classical (“hard”) geometric predicates must be developed.
What are the consequences of such a computational paradigm?
* scope of computational geometry would be vastly broadened
* some unsolvable problems in the Real RAM model becomes possible
* soft algorithms are not only implementable but can be practical

One challenge is to revisit classical problems of computational geometry from this new view point. Another challenge is to produce complexity analysis of such algorithms.

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