Blended Hermite Interpolants

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Abstract

In [6] B-spline technique was used for blending of Lagrange interpolants. In this paper we generalize this idea replacing Lagrange by Hermite interpolants. The generated subspline $\mathbf{b}(t)$ interpolates the Hermite input data consisting of parameter values t_i and corresponding derivatives $\mathbf{a}_{i,j}$, $j = 0, \ldots, \alpha_i - 1$ and is called blended Hermite interpolant (BHI). It has local control, is connected in affinely invariant way with the input and consists of integral (polynomial) segments of degree $2 \cdot k - 1$, where $k - 1 \geq max\{\alpha_i\} - 1$ denotes the degree of the B-spline basis functions used for the blending. This method automatically generates one of the possible interpolating subsplines of class C^{k-1} with the advantage that no additional input data is necessary.

Key words: Hermite interpolation, B-splines basis functions

1 Hermite Interpolation

Hermite interpolation data consists of real parameter values $t_0 < \ldots < t_i < \ldots < t_l$ and corresponding points and derivatives (vectors in $\mathbb{R}^d$, $d \in \mathbb{N}$) $\mathbf{a}_{i,j}$, where $i = 0, \ldots, l$ and $j = 0, \ldots, \alpha_i - 1$ with $l \in \mathbb{N}_0$, $\alpha_i \in \mathbb{N}$. It is well known [1], that there is exactly one integral (polynomial) curve $\mathbf{h}(t)$ with polynomial degree less than or equal to $n := \sum_{i=0}^{l} \alpha_i - 1$ satisfying [2]

$$\mathbf{h}^{(j)}(t_i) = \mathbf{a}_{i,j} \quad \text{for all} \quad i = 0, \ldots, l \quad \text{and} \quad j = 0, \ldots, \alpha_i - 1. \tag{1}$$

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[1] See [5, pages 4–11]

[2] Derivatives will either be indicated by superscripts in parentheses or in the form $\frac{d^j}{(dt)^j}$.

A parametric representation of this Hermite interpolant $\mathbf{h}(t)$ is given by[3]

$$\mathbf{h}(t) = \sum_{i=0}^{l} \sum_{j=0}^{\alpha_i - 1} \mathbf{a}_{i,j} \sum_{m=0}^{\alpha_i - j - 1} \frac{1}{m! \cdot j!} \left[\frac{\mathrm{d}^m}{(\mathrm{d}t)^m} \left(\frac{(t - t_i)^{\alpha_i}}{p(t)} \right) \right]_{t_i} \frac{p(t)}{(t - t_i)^{\alpha_i - j - m}}, \quad (2)$$

where

$$p(t) := \prod_{i=0}^{l} (t - t_i)^{\alpha_i}. \quad (3)$$

In the special case of $\alpha_0 = \ldots = \alpha_l = 1$ eq. (2) yields the ordinary Lagrange interpolant

$$\mathbf{l}(t) = \sum_{i=0}^{l} \mathbf{a}_{i,0} \cdot \frac{p(t)}{(t - t_i) \cdot p^{(1)}(t_i)}, \quad (4)$$

whereas in case of $l = 0$ one obtains the Taylor expansion of order $\alpha_0 - 1$ at t_0:

$$\mathbf{t}_{0,\alpha_0 - 1}(t) = \sum_{j=0}^{\alpha_0 - 1} \mathbf{a}_{0,j} \cdot \frac{1}{j!} \cdot (t - t_0)^j. \quad (5)$$

Fig. 1. Definition of the knots u_i.

It is convenient to set (see figure 1)

$$\left.\begin{array}{lll} u_0 = \ldots = u_{\alpha_0 - 1} & := t_0 \\ u_{\alpha_0} = \ldots = u_{\alpha_0 + \alpha_1 - 1} & := t_1 \\ \vdots & \vdots \\ u_{\alpha_0 + \ldots + \alpha_{l-1}} = \ldots = u_n & := t_l \end{array}\right\} \quad (6)$$

and to denote the corresponding Hermite interpolant by $\mathbf{h}_{0,n}(t)$.

[3] See [1, page 121]

Let moreover k denote a natural number $\geq max\{\alpha_i\}$. Then $\mathbf{h}_{\beta,k}(t)$ shall denote the *partial Hermite interpolant of degree* k in the following sense. Beginning with u_β the next k knots are used to define $\mathbf{h}_{\beta,k}(t)$: According to (6) the knot sequence $(u_\beta, \ldots, u_{\beta+k})$ can be written as

$$(u_\beta, \ldots, u_{\beta+k}) = (\underbrace{t_{i_1}, \ldots, t_{i_1}}_{r_1 \text{ times}}, \ldots, \underbrace{t_{i_d}, \ldots, t_{i_d}}_{r_d \text{ times}}) \tag{7}$$

with $t_{i_1} < \ldots < t_{i_d}$ and $r_\nu \leq \alpha_{i_\nu}$ for $\nu = 1, \ldots, d$. We have for the corresponding partial Hermite interpolant $\mathbf{h}_{\beta,k}(t)$ of degree k

$$\mathbf{h}^{(j)}_{\beta,k}(t_{i_\nu}) = \mathbf{a}_{i_\nu,j} \tag{8}$$

for all $j = 0, \ldots, r_\nu - 1$ and $\nu = 1, \ldots, d$.

2 Blending of Hermite interpolants via B-spline-basis functions

The intention of this section is to construct for a given natural number $k \geq \max\{\alpha_i\}$ a subspline $\mathbf{b}(t)$ which interpolates the given Hermite data t_i, $\mathbf{a}_{i,j}$ (see section 1) with polynomials segments of degree $2 \cdot k - 1$ and smoothness $k - 1$ at the knots. The existence of such a subspline is apparent from the following simple consideration: We could add $k - \alpha_i$ arbitrarily chosen derivative vectors $\mathbf{a}_{i,j}$, $j = \alpha_i, \ldots, k - 1$ at any knot t_i with $\alpha_i < k$. Then there exist unique partial Hermite interpolants $\mathbf{h}_{i,2k-1}(t)$ satisfying the Hermite data $\mathbf{a}_{i,j}$ and $\mathbf{a}_{i+1,j}$, $j = 0, \ldots, k - 1$ belonging to two adjacent knots t_i and t_{i+1}. Using them as spline segments, yields a subspline with the required properties (see also [7]).

But here we will point out a different way. We will generate a subspline by blending partial Hermite interpolants (of degree k) by B-spline basis functions of degree $k-1$. Surprisingly, this directly leads to a subspline (of degree $2 \cdot k - 1$) which interpolates the given input. This method does not need additional derivative vectors.

For a given array $U := (u_0 \leq u_1 \leq \ldots \leq u_n)$ of parameter-knots (normalized) B-spline-basis functions $N_{\beta,k}(t)$ are defined recursively:

$$j = 1: \quad N_{\beta,1}(t) := \begin{cases} 1 \text{ for } t \in [u_\beta, u_{\beta+1}) \\ 0 \text{ else} \end{cases} \tag{9}$$

$$j = 2, \dots, k: \quad N_{\beta,j}(t) := \frac{t - u_\beta}{u_{\beta+j-1} - u_\beta} N_{\beta,j-1}(t) + \frac{u_{\beta+j} - t}{u_{\beta+j} - u_{\beta+1}} N_{\beta+1,j-1}(t)$$

for $\beta = 0, \dots, n-k$ with $\frac{0}{0} := 0$ and $N_{n-k,1}(u_{n-k+1}) := 1$.

The functions $N_{\beta,k}(t)$ have the following properties: [4]

Trivially the B-spline basis functions $N_{\beta,k}(t)$ are C^∞ on each intervall (u_m, u_{m+1}), $t_i = u_m \neq u_{m+1} = t_{i+1}$. In such an interval only the functions $N_{m-k+1,k}(t), \dots, N_{m,k}(t)$ are different from zero. In the interval $[u_m, u_{m+1})$ these functions are identical with polynomials $\overline{N}_{m-k+1,k}(t), \dots, \overline{N}_{m,k}(t)$ of degree $\leq k-1$. At the parameter knots we have: If $u_{m-\alpha_i+1} = \dots = u_m = t_i$, for $\alpha_i \leq k$ then the functions $N_{m-k+1,k}(t), \dots, N_{m,k}(t)$ are $k - \alpha_i - 1$ times differentiable at t_i.

The functions $N_{\beta,k}(t)$ form a partition of unity - therefore we have:

$$\frac{d^\gamma}{(dt)^\gamma} \left[\sum_{\beta=0}^{n-k} N_{\beta,k}(t) \right] \stackrel{t}{\equiv} \delta_{0,\gamma} \tag{10}$$

with the Kronecker symbol $\delta_{0,0} := 1$ and $\delta_{0,\gamma} := 0$ else.

Let now Hermite interpolation data t_i, $\mathbf{a}_{i,j}$, $i = 0, \dots, l$, $j = 0, \dots, \alpha_i - 1$ be given. Let again $n := \sum_{i=0}^{l} \alpha_i - 1$ and let moreover k denote a natural number with [5]

$$\max\{\alpha_i\} \leq k < \frac{n+2}{2}. \tag{11}$$

According to section 1 the *partial Hermite interpolants* $\mathbf{h}_{\beta,k}(t)$ *of degree* k are well defined for $\beta = 0, \dots, n-k$. Now we define a curve as follows:

$$\mathbf{b}(t) := \sum_{\beta=0}^{n-k} N_{\beta,k}(t) \cdot \mathbf{h}_{\beta,k}(t), \quad t \in [u_{k-1}, u_{n-k+1}], \quad k \geq 2. \tag{12}$$

Here the "coefficients" of the B-spline-basis functions are the partial Hermite interpolants $\mathbf{h}_{\beta,k}(t)$. This curve will be called *blended Hermite interpolant (BHI)*.

To show some essential properties of BHI-curves we need

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[4] See [4, pp. 162]

[5] The second inequality in (11) guarantees that the support interval of the B-spline basis functions is not empty.

Lemma 1 *Let $u_{m-\alpha_i+1} = \ldots = u_m = t_i \neq u_{m+1}$; then for $\beta = m - \alpha_i + 1, \ldots, m$: $(t - t_i)^{k-1-m+\beta}$ is a factor of $\overline{N}_{\beta,k}(t)$.*

This can be easily verified by making use of the recursive definition (9) of the B-spline basis functions.

If $\alpha_i < k$ (**case A**) we get as a conclusion of this lemma:

$$\begin{array}{llllll} 0 = \overline{N}^{(0)}_{m-\alpha_i+1,k} = \ldots = \overline{N}^{(k-\alpha_i-1)}_{m-\alpha_i+1,k} & & & \\ 0 = \overline{N}^{(0)}_{m-\alpha_i+2,k} = \ldots = \overline{N}^{(k-\alpha_i-1)}_{m-\alpha_i+2,k} & = \overline{N}^{(k-\alpha_i)}_{m-\alpha_i+2,k} & & \\ \vdots \qquad\qquad \vdots & & \ddots & \\ 0 = \overline{N}^{(0)}_{m,k} = \ldots = \overline{N}^{(k-\alpha_i-1)}_{m,k} & = \overline{N}^{(k-\alpha_i)}_{m,k} & = \ldots = \overline{N}^{(k-2)}_{m,k}, \end{array} \tag{13}$$

whereas in case of $\alpha_i = k$ (**case B**) we get

$$\begin{array}{llll} 0 = \overline{N}^{(0)}_{m-k+2,k} & & \\ 0 = \overline{N}^{(0)}_{m-k+3,k} = \overline{N}^{(1)}_{m-k+3,k} & & \\ \vdots & \ddots & \\ 0 = \overline{N}^{(0)}_{m,k} = \ldots & = \overline{N}^{(k-2)}_{m,k}. \end{array} \tag{14}$$

Additionally, at $t = t_i$ the basic partial Hermite interpolants satisfy

$$\begin{array}{llll} \mathbf{h}^{(\alpha_i-1)}_{m-k+1,k} = \mathbf{a}_{i,\alpha_i-1}, & \mathbf{h}^{(\alpha_i-2)}_{m-k+1,k} = \mathbf{a}_{i,\alpha_i-2}, & \ldots, & \mathbf{h}^{(0)}_{m-k+1,k} = \mathbf{a}_{i,0} \\ \vdots & & & \vdots \\ \mathbf{h}^{(\alpha_i-1)}_{m-\alpha_i+1,k} = \mathbf{a}_{i,\alpha_i-1}, & \mathbf{h}^{(\alpha_i-2)}_{m-\alpha_i+1,k} = \mathbf{a}_{i,\alpha_i-2}, & \ldots, & \mathbf{h}^{(0)}_{m-\alpha_i+1,k} = \mathbf{a}_{i,0} \\ & \mathbf{h}^{(\alpha_i-2)}_{m-\alpha_i+2,k} = \mathbf{a}_{i,\alpha_i-2}, & \ldots, & \mathbf{h}^{(0)}_{m-\alpha_i+2,k} = \mathbf{a}_{i,0} \\ & & \ddots & \vdots \\ & & & \mathbf{h}^{(0)}_{m,k} = \mathbf{a}_{i,0} \end{array} \tag{15}$$

Theorem 2 (a) *The blended Hermite interpolant* $\mathbf{b}(t)$ *belongs to the continuity class* C^{k-1}.
(b) $\mathbf{b}^{(j)}(t_i) = \mathbf{a}_{i,j}$ *for* $i = 0, \ldots, l$ *with* $u_{k-1} \leq t_i \leq u_{n-k+1}$, $j = 0, \ldots, \alpha_i - 1$.
(c) *The polynomial degree of the segments of* $\mathbf{b}(t)$ *is* $\leq 2 \cdot k - 1$.

Proof

(a) Let $t_{i-1} = u_{m-\alpha_i} < u_{m-\alpha_i+1} = \ldots = u_m = t_i < u_{m+1} = t_{i+1}$ (see figure 1). If $j < k - \alpha_i$ no proof is necessary since the functions $N_{\beta,k}(t)$ are $k - \alpha_i - 1$

- times differentiable at t_i. Let now $k-\alpha_i \leq j \leq k-1$. Then using product rule the j^{th} right-hand derivative $\mathbf{b}_+^{(j)}(t_i)$ of $\mathbf{b}(t)$ at t_i is

$$\mathbf{b}_+^{(j)}(t_i) = \sum_{\gamma=0}^{j} \binom{j}{\gamma} \sum_{\beta=0}^{n-k} N_{\beta,k,+}^{(\gamma)}(t_i) \cdot \mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i).$$

Like above we have two consider two different cases: $\alpha_i < k$ (case A) and $\alpha_i = k$ (case B).

Case A ($\alpha_i < k$). Here we can split this sum into two parts:

$$\mathbf{b}_+^{(j)}(t_i) = \overbrace{\sum_{\gamma=0}^{k-\alpha_i-1} \binom{j}{\gamma} \sum_{\beta=0}^{n-k} N_{\beta,k,+}^{(\gamma)}(t_i) \cdot \mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i)}^{(*)}$$
$$+ \underbrace{\sum_{\gamma=k-\alpha_i}^{j} \binom{j}{\gamma} \sum_{\beta=0}^{n-k} N_{\beta,k,+}^{(\gamma)}(t_i) \cdot \mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i)}_{(**)}.$$

Since the $N_{\beta,k}(t)$ are $k-\alpha_i-1$ - times differentiable at t_i the first partial sum $(*)$ is identical with its counterpart in the left-hand derivative $\mathbf{b}_-^{(j)}(t_i)$ of $\mathbf{b}(t)$ at t_i.

In $(**)$ the range of the summation index β can be restricted to $m-k+1, \ldots, m$ since the other B-spline functions vanish identically on the interval $[t_i, t_{i+1})$. Moreover the remaining functions $N_{\beta,k}(t)$ are identical with the polynomials $\overline{N}_{\beta,k}(t)$ on $[t_i, t_{i+1})$:

$$(**) = \sum_{\gamma=k-\alpha_i}^{j} \binom{j}{\gamma} \sum_{\beta=m-k+1}^{m} \overline{N}_{\beta,k}^{(\gamma)}(t_i) \cdot \mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i),$$

The upper summation bound m of the second sum in $(**)$ can be replaced by $m-k+\gamma+1$ due to (13). Then using (15) one can substitute $\mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i) = \mathbf{a}_{i,j-\gamma}$, hence

$$(**) = \sum_{\gamma=k-\alpha_i}^{j} \binom{j}{\gamma} \cdot \mathbf{a}_{i,j-\gamma} \cdot \sum_{\beta=m-k+1}^{m-k+\gamma+1} \overline{N}_{\beta,k}^{(\gamma)}(t_i)$$
$$= \sum_{\gamma=k-\alpha_i}^{j} \binom{j}{\gamma} \cdot \mathbf{a}_{i,j-\gamma} \cdot \underbrace{\frac{d^\gamma}{(dt)^\gamma} \left[\sum_{\beta=0}^{n-k} N_{\beta,k}(t) \right]_{t=t_i}}_{=0, \text{ due to } (10)} = \mathbf{o}.$$

In a completely analogous way we can show that the partial sum $\gamma = k - \alpha_i, \ldots, j$ of the left-hand derivative $\mathbf{b}_-^{(j)}(t_i)$ vanishes. This means that $\mathbf{b}_+^{(j)}(t_i)$ and $\mathbf{b}_-^{(j)}(t_i)$ are identical in case A. Hence also for $j = k - \alpha_i, \ldots, k-1$ the j^{th} derivatives of $\mathbf{b}(t)$ at t_i do exist in this case.

Case B ($\alpha_i = k$) Like in case A we can reduce the range of the summation index β to $m-k+1, \ldots, m$ and replace the right-hand derivatives $N_{\beta,k,+}^{(\gamma)}(t_i)$ by the derivatives $\overline{N}_{\beta,k}^{(\gamma)}(t_i)$ of the corresponding polynomials:

$$\mathbf{b}_+^{(j)}(t_i) = \sum_{\gamma=0}^{j} \binom{j}{\gamma} \sum_{\beta=m-k+1}^{m} \overline{N}_{\beta,k}^{(\gamma)}(t_i) \cdot \mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i),$$

According to (14) the upper summation bound m in the second sum can again be reduced to $m-k+\gamma+1$ and then due to (15) $\mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i)$ can be replaced by $\mathbf{a}_{i,j-\gamma}$:

$$\begin{aligned}
\mathbf{b}_+^{(j)}(t_i) &= \sum_{\gamma=0}^{j} \binom{j}{\gamma} \cdot \mathbf{a}_{i,j-\gamma} \cdot \sum_{\beta=m-k+1}^{m-k+\gamma+1} \overline{N}_{\beta,k}^{(\gamma)}(t_i) \\
&= \sum_{\gamma=0}^{j} \binom{j}{\gamma} \cdot \mathbf{a}_{i,j-\gamma} \cdot \underbrace{\frac{d^\gamma}{(dt)^\gamma} \left[\sum_{\beta=0}^{n-k} N_{\beta,k}(t) \right]_{t=t_i}}_{=\delta_{0,\gamma},\text{ due to (10)}} = \mathbf{a}_{i,j}.
\end{aligned}$$

By an analogous consideration we can show that the j^{th} left-hand derivative of $\mathbf{b}(t)$ at t_i is equal to $\mathbf{a}_{i,j}$ if $\alpha_i = k$:

$$\mathbf{b}_-^{(j)}(t_i) = \mathbf{a}_{i,j}.$$

Hence, in case B too the j^{th} derivative of $\mathbf{b}(t)$ at t_i exists for all in $j = 0, \ldots, k-1$. This finishes the proof of (a).

Additionally we have found out that for all $j = 0, \ldots, k-1$:

$$\mathbf{b}^{(j)}(t_i) = \sum_{\gamma=0}^{\min\{j,k-\alpha_i-1\}} \binom{j}{\gamma} \sum_{\beta=m-k+1}^{m} N_{\beta,k}^{(\gamma)}(t_i) \cdot \mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i) \text{ if } \alpha_i < k, \tag{16}$$

$$\mathbf{b}^{(j)}(t_i) = \mathbf{a}_{i,j} \text{ if } \alpha_i = k. \tag{17}$$

(b) Let now $j = 0, \ldots, \alpha_i - 1$.

Case A ($\alpha_i < k$): The derivatives $\mathbf{b}^{(j)}(t_i)$ (exist and) are computed according to (16). Since for the first summation index γ the inequality $\gamma \leq k - \alpha_i - 1$ holds, the upper summation bound m of the second sum can be reduced to $m-\alpha_i$ by using (13). Due to (15) we moreover have for $\beta = m-k+1, \ldots, m-\alpha_i$, $\gamma \leq j \leq \alpha_i - 1$: $\mathbf{h}_{\beta,k}^{(j-\gamma)}(t_i) = \mathbf{a}_{i,j-\gamma}$. Hence

$$\begin{aligned}\mathbf{b}^{(j)}(t_i) &= \sum_{\gamma=0}^{\min\{j,k-\alpha_i-1\}} \binom{j}{\gamma} \cdot \mathbf{a}_{i,j-\gamma} \cdot \sum_{\beta=m-k+1}^{m-\alpha_i} N_{\beta,k}^{(\gamma)}(t_i) \\ &= \sum_{\gamma=0}^{\min\{j,k-\alpha_i-1\}} \binom{j}{\gamma} \cdot \mathbf{a}_{i,j-\gamma} \cdot \underbrace{\frac{d^\gamma}{(dt)^\gamma}\left[\sum_{\beta=0}^{n-k} N_{\beta,k}(t)\right]_{t=t_i}}_{=\delta_{0,\gamma},\text{ due to (10)}} = \mathbf{a}_{i,j}.\end{aligned}$$

In case B ($\alpha_i = k$) the statement is true due to (17).

This finishes the proof of (b).

(c) The statement is trivial due to the generation of the subspline. □

As announced at the beginning of this section the construction of the BHI does not need any further input of derivative vectors. For any knot t_i with $\alpha_i < k$ the additional derivatives $\mathbf{a}_{i,\alpha_i} := \mathbf{b}^{(\alpha_i)}(t_i), \ldots, \mathbf{a}_{i,k-1} := \mathbf{b}^{(k-1)}(t_i)$ can be computed via eq. (16).

3 Additional properties of blended Hermite splines

Affine invariance.
As the computation of the BHI can be done by linear subdivision – using first the divided differences scheme[6] for the points on the partial Hermite interpolants and then Cox-deBoor-algorithm for the B-spline blending – it is clear that these curves are connected with their input vectors $\mathbf{a}_{i,j}$ and the knot sequence in an affinely invariant way: Applying an affine mapping τ on the vectors $\mathbf{a}_{i,j}$ and then constructing the corresponding BHI gives the same result as first constructing the BHI and then applying τ.

Local control.
As the B-spline-basic functions and the partial Hermite interpolants used for the definition have local control, our BHI-subspline has this property too.

[6] see [5, pages 16–21]

Begin-to-end interpolation.
In order to interpolate also the data t_i, $\alpha_{i,j}$ with $t_i < u_{k-1}$ and those with $t_i > u_{n-k+1}$ one has – like in the case of ordinary B-splines – to add knots at the beginning and at the end of the knot vector U and multiply count the Hermite splines $\mathbf{b}_{0,k}(t)$ and $\mathbf{b}_{n-k,k}(t)$. For instance to achieve interpolation at the beginning one has to add $k - \alpha_0$ knots so that one has exactly k identical knots at the beginning. In addition one has to count the first partial Hermite interpolant $\mathbf{b}_{0,k}(t)$ $k-\alpha_0+1$ times. Analogously for interpolation of the data at the end ($t_i > u_{n-k+1}$) one has to extend the knot vector U to k identical knots at the end and count $\mathbf{b}_{n-k,k}(t)$ $k-\alpha_l+1$ times. An example for begin-to-end interpolation for different values of k is shown in figure 2.

Closed Interpolation.
It is also simple to construct a closed BHI: One just has to "overlap" the existing data as in case of ordinary B-splines.[7] Figure 3 shows an example for closed interpolation, varying the result by changing the lenghts of the first derivative vectors in two of the points. k was chosen to be 4.

Special cases.
If $\alpha_i = k$ for $i = 0, \ldots, l$ we get a BHI $\mathbf{b}(t)$ whose polynomial segments are the Hermite interpolants belonging to the input data t_i, $\mathbf{a}_{i,j}$ and t_{i+1}, $\mathbf{a}_{i+1,j}$ for $j = 0, \ldots, k$.

If $\alpha_i = 1$ for $i = 0, \ldots, l$ one obtains a blending of Lagrange interpolants – this is the case treated in [6].

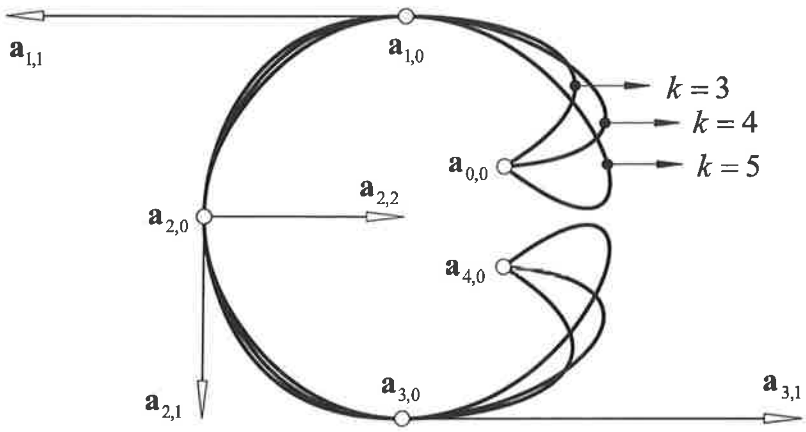

Fig. 2. Example for begin-to-end interpolation for different values of k ($l = 4$, $\alpha_0 = 1$, $\alpha_1 = 2$, $\alpha_2 = 3$, $\alpha_3 = 2$, $\alpha_4 = 1$).

[7] See [4].

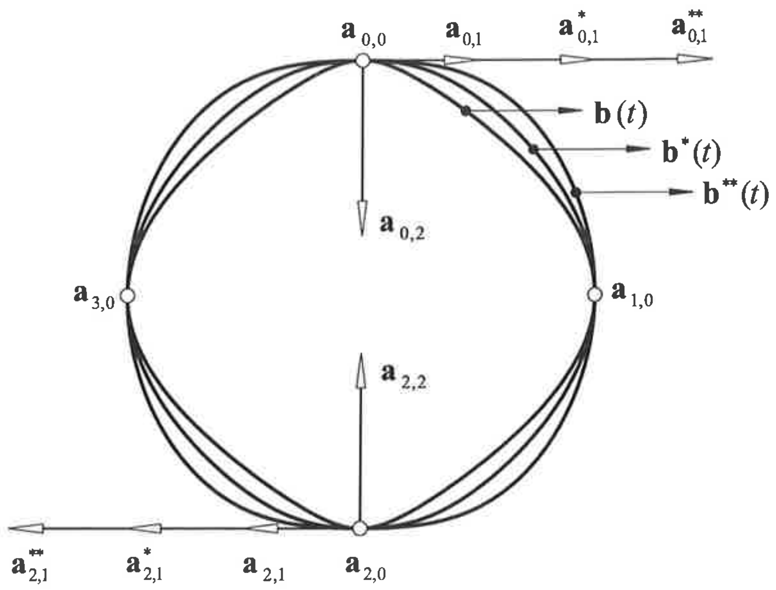

Fig. 3. Example for a closed interpolation for varying derivative vectors $\mathbf{a}_{0,1}$, $\mathbf{a}_{2,1}$.

4 Conclusions

We defined a new interpolating subspline for Hermite interpolation data. The *blended Hermite interpolant* (BHI) was generated by blending partial Hermite interpolants by the normalized B-spline basic functions. The segments are integral curves of degree $2 \cdot k - 1$, which are of class C^{k-1} at the knots t_i. At these knots the input data are interpolated. All is defined in an affinely invariant way. According to the B-spline curve scheme it is easy to generate an open or closed BHI.

The idea of the BHI could be seen as a limit consideration of the paper [6]: The Langrangian input of that paper is changed into the Hermite input. Then B-spline blending is performed. Then it is easy to sweep to integral B-spline curves. In this cases the spline only belongs to the class C^{k-2}, of course.

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