Tiling freeform shapes with straight panels: Algorithmic methods.

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Abstract. *This paper shows design studies with bent panels which are originally rectangular or at least approximately rectangular. Based on recent results obtained in the geometry processing community, we algorithmically approach the questions of an exact rectangular shape of panels; of watertightness of the resulting paneling; and of the panel shapes being achievable by pure bending. We conclude the paper with an analysis of stress and strain in bent and twisted panels.*

1 Introduction

This paper is concerned with panels of wood or metal, which are mounted on freeform surfaces, and which in their flat state are rectangular (or can at least be cut from rectangles). Figure 1 gives an impression of the kind of example we have in mind. In particular we deal with a mathematical formulation and algorithmic approach to this topic. Such patterns occur in the cladding of general freeform (double curved) shapes, for instance applied to interior surfaces. An experimental example, which is taken from [Spuybroek 2004], is shown by Figure 2.

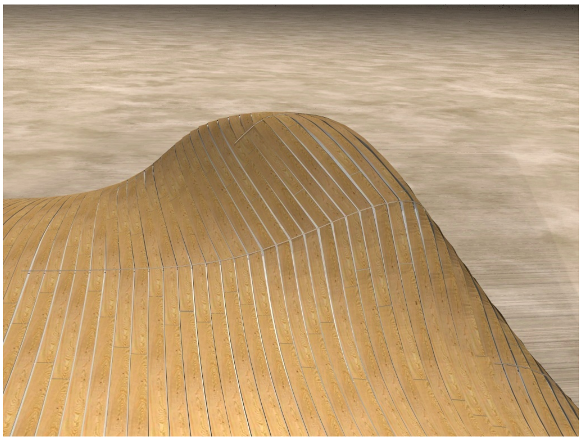
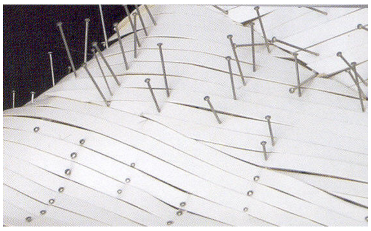


Figure 1: This image gives an impression of rectangular panels mounted on a freeform shape in an optimized pattern: Gaps are deliberately left open in order to illustrate how little the panel widths would have to be modified in order to achieve watertight paneling (cf. Figures 5, 14).



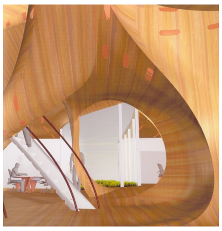


Figure 2: Experimental cladding using paper strips (left) results in an office space design by NOX Architects (right, see [Spuybroek 2004]).

In order to understand the *geometry* which governs the behaviour of panels, we discuss the various issues which arise when trying to cover freeform shapes with rectangular panels. There are several properties of the resulting patterns which one would like to have — each property being derived from practical considerations and giving rise to its own mathematical theory. Unfortunately only in rare instances we can have all of these properties at the same time. Usually a compromise will have to be found.

The geodesic property. Long and thin panels easily bend about their weak axis and may twist a bit, but for all practical purposes they do not bend about their strong axis. This translates into the mathematical statement that such a panel, if laid onto a surface, follows a *geodesic curve*. These curves are equally characterized by having zero geodesic curvature, and by being the shortest curves which connect different points of a surface. For more information on geodesics, the reader is referred to textbooks of differential geometry such as [do Carmo 1976].

The constant width property. We think of panels whose original, unfolded shape is a rectangle (see Figure 2, where those panels are represented as strips of paper). Only special shapes can be covered by such panels in a seamless and non-overlapping way: basically the only way in which this can happen is that the entire surface is itself a *developable surface*. For all other surfaces, assuming we have no gaps or overlaps, panels are not exactly rectangular when unfolded. In any case it is very important for the practical fabrication of such panels that they can be cut from a rectangular shape without too much waste. Mathematically this leads us to the requirement that the geodesic curves which guide the panels must have approximately *constant distance* from their neighbour curves.

The developable (or 'pure bending') property. The process of bending a surface changes the distances of points only by a very small amount, if those distances are measured inside the surface. A certain amount of twisting, as opposed to pure bending, is present in the applications we have in mind. While the previous two properties actively influence all our algorithmic approaches, the developable property is present in only one of them.

The issues discussed above lead to the following questions:

Problem statement 1. *We look for a system of geodesic curves in a freeform surface which are at approximately constant distance from their neighbours, and which can serve as guiding curves for the bending of rectangular wooden panels. Those panels are to cover the surface with only small gaps and no overlaps.*

Problem statement 2. *We look for a system of geodesic curves in a freeform surface which serve as the boundaries of wooden panels whose development is approximately straight and which can be cut from a rectangular shape. Those panels are to cover the surface without gaps.*

Previous work. Questions of this kind and generally the layout of geodesic patterns on surfaces have recently attracted great interest in the geometry processing community. [Kahlert et al. 2010] study the tiling of a surface by strips of controlled width which are bounded by geodesics. They employ an evolution method, starting from a single geodesic and proceeding from there until the surface under consideration is exhausted. [Pottmann et al. 2010] investigate general and multiple patterns of geodesics on freeform surfaces. They propose a mixture of methods (evolution, level set, geodesic vector fields), and it is that paper which our work is mainly based on.

— *Related work: Computing geodesics.* The theory of geodesics is found in textbooks of differential geometry such as [do Carmo 1976]. For computational purposes, shapes are represented as triangle meshes, and their geodesics are represented as polylines in meshes which are the shortest connections between points. That definition is usually sufficient but may lead to ambiguities which can be resolved by the concept of "straightest geodesics" [Polthier and Schmies 1998] which we use in our algorithms. Finding the truly shortest geodesic paths requires the computation of distance fields, for which several efficient algorithms have been developed, see for instance [Chen and Han 1996] or [Kimmel and Sethian 1998], or the later paper [Surazhsky et al. 2005].

— *Related work: Timber constructions and geodesics.* Geodesic curves have made their appearance in freeform architecture in another context, namely in the supporting structures of curved shells. [Pirazzi and Weinand 2006] show the design of freeform timber rib shells which are composed of screw-laminated beams. If such beams are considered as curves in the surface they support, then they have zero geodesic curvature, i.e., they are geodesics.

— *Related work: Rationalization of freeform surfaces by developable strips.* Early research on the cladding of freeform surfaces with developable panels evolved from the architecture of F. Gehry [Shelden 2002]. That work however does not deal with the decomposition of general shapes into developable strips, which problem was algorithmically solved by [Pottmann et al. 2008]. Already in that paper a notion of *geodesic strips* was defined: we discuss them later. The authors emphasize that in general any decomposition of a surface into developable strips must be such that the strip boundaries stay away from the *asymptotic directions* in the saddle-shaped regions of the surface. Differential-geometric issues of that kind will also be present in our work.

2 The design of patterns of geodesics.

As a prerequisite for solving Problems 1 and 2 we first discuss patterns of geodesic curves in surfaces and methods to create them. Subsequent sections translate the geometric information stored in these curve patterns into actual paneling.

Let us rehearse the various properties of geodesics: They are the curves in a surface with zero geodesic (i.e., sideways) curvature. They are uniquely determined by an initial point and tangent. Mathematically, if a point $p(t)$ is moving in time t with unit speed, then it moves along a geodesic if and only if the second derivative vector $p''(t)$ remains orthogonal to the surface. Also the shortest connections between points in the surface are geodesics.

2.1 Design by parallel transport.

In this section we describe how to find patterns of geodesics where either the maximum distance or the minimum distance between adjacent curves occurs at a prescribed location. This method is briefly described by [Pottmann et al. 2010].

Differential geometry knows the notion of *parallel transport* of a vector V along a curve s contained in a surface. It means moving that vector along s such that it remains tangent to the surface, but such that it changes as little as possible (i.e., $\|V'(t)\|$ is minimal). It is known that the length of that vector remains unchanged [do Carmo 1976]. If, for computational purposes, a surface is represented as a mesh and a curve is represented as a polyline with vertices $P_0, P_1, P_2, \ldots$, we emulate parallel transport along that polyline by a simple step-by-step procedure explained in Figure 3.

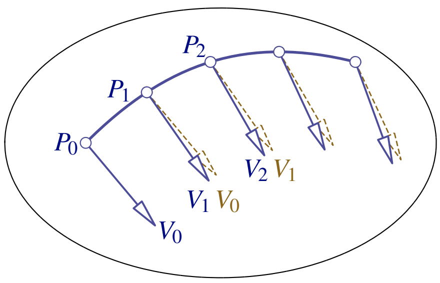


Figure 3: Parallel transport of a vector V_0 attached to the vertex P_0 along the polyline $P_0P_1P_2\ldots$ is algorithmically realized as follows: V_i is found by orthogonal projection of V_{i-1} onto the tangent plane of P_i, and subsequent re-normalizing.

Parallel transport has the following property relevant to the design of patterns of geodesics: Suppose a curve is sampled at points $P_0, P_1, \ldots$ as shown by Figure 3 and that geodesic parallel transport yields vectors $V_0, V_1, \ldots$ attached to these points. Consider the geodesic rays which emanate from the point P_i in direction V_i and $-V_i$ (two such rays together make one unbroken geodesic). Figure 4 shows an example of that. *Then extremal distances between adjacent geodesics occur near the chosen curve*.

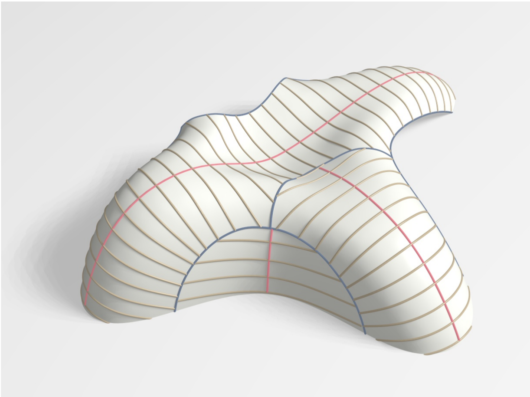


Figure 4: Designing a sequence of geodesics by choosing the locus (red) of minimum distance or maximum distance between neighbours. This is done by the *parallel transport method*. In this particular example, the method is applied not to the entire surface, but to previously selected patches.

2.2 Design by evolution and by segmentation

We first briefly rehearse the evolution method proposed by [Pottmann et al. 2010]. Starting from a *source geodesic* somewhere in the given surface, we evolve a pattern of geodesics, iteratively computing 'next' geodesics, each having approximately constant distance from its predecessor. This is not possible in an exact way on general surfaces, and if the deviation from a predefined width becomes too great one might to have to introduce breakpoints and proceed further with piecewise-geodesic curves. Figure 5 illustrates how this procedure works; for algorithmic details we refer to [Pottmann et al. 2010].

Another method employed by [Pottmann et al. 2010] is based on the concept of piecewise-geodesic vector fields. We cannot attempt to describe it here, but we mention that it performs *segmentation* of the given freeform shape into parts which are nicely coverable by a pattern of geodesic lines. Both Figure 4 and Figure 6 show an example of this. For Figure 4, the single patches which emerge after segmentation have been treated with the parallel transport method. For Figure 6, the evolution method has been used.

3 Panels from curve patterns.

Panels as we consider them are originally flat, and when mounted onto a surface they are bent (and twisted if necessary). We investigate two different ways of mathematical representation of such panels: One which produces almost exactly developable shapes which are achievable by pure bending, and another method where we check for the amount of twisting only afterwards. Unfortunately the first method is hindered by obstructions of a fundamental nature.

The exact relation between the ideal design surface Φ to be covered by the panels on the one hand, and the panels themselves on the other hand, needs clarification. One possibility is that we model the panel surfaces so that they are tangentially circumscribed to Φ along given geodesic curves; and this is what we do.

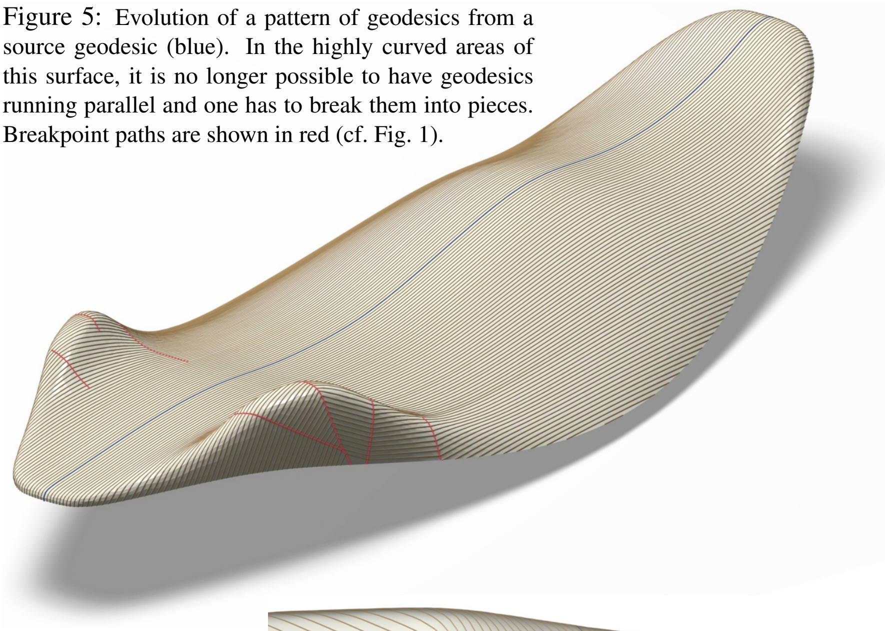


Figure 5: Evolution of a pattern of geodesics from a source geodesic (blue). In the highly curved areas of this surface, it is no longer possible to have geodesics running parallel and one has to break them into pieces. Breakpoint paths are shown in red (cf. Fig. 1).

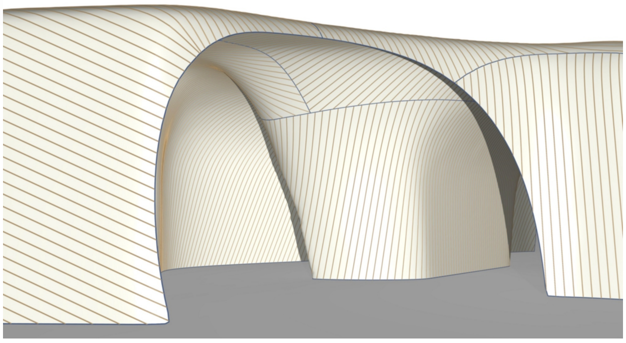


Figure 6: Segmenting a surface into pieces which can nicely be covered by a sequence of geodesic lines. For the covering, the evolution method was employed.

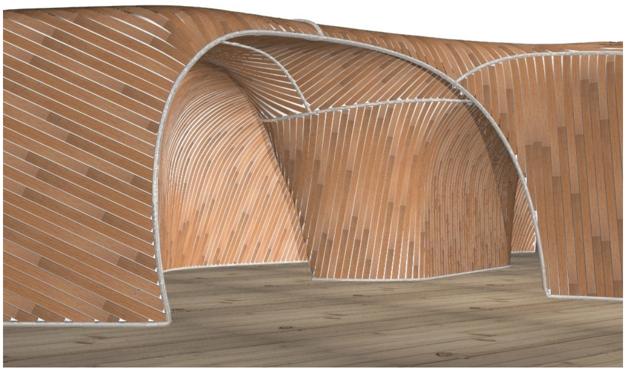


Figure 7: This design with bent rectangular panels is based on Figure 6.

Another idea is that the panel surfaces are *inscribed* into the design surface. For instance we could connect two neighbouring geodesics by a developable surface which is subsequently used for the panel. Algorithmically this is not easy [Rose et al. 2007] and anyway we would rather have a geodesic running in the center of the panel (which is achieved with the idea of circumscribed panels).

3.1 Panels with pure bending: the tangent developable method.

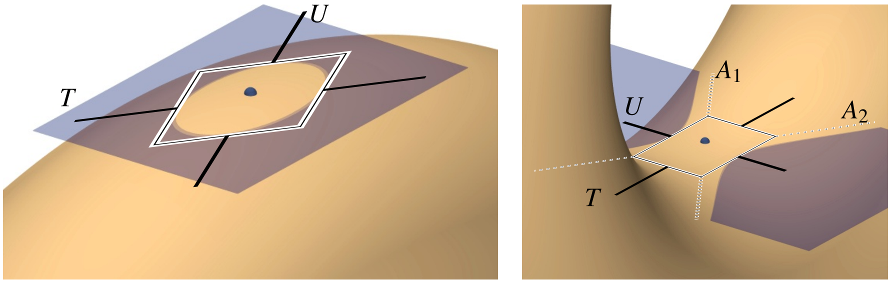


Figure 8: Illustration of asymptotic directions A_1, A_2 and conjugate directions T, U: Parallel translation of a tangent plane (blue) by a small amount and intersection with the surface yields a curve which approximates a conic section (the *Dupin indicatrix*). In negatively curved areas this is a hyperbola, whose asymptotes A_1, A_2 define the *asymptotic directions*. Any parallelogram tangentially circumscribed to the indicatrix defines two conjugate tangents T, U. It is known that A_1, A_2 are diagonals of any such parallelogram. Obviously choosing T determines U. For both figures, the base surface is a torus.

For smooth surfaces the notion of *conjugate tangents* is defined; they are explained by Figure 8. Mathematically vectors v, w which are expressed in a coordinate system whose basis are principal curvature vectors are conjugate, if and only if $v^T \operatorname{diag}(\kappa_1, \kappa_2) w = 0$, where κ_1, κ_2 are the principal curvatures. Algorithmically, curvatures and conjugate tangents can be computed from triangle meshes by well known methods of geometry processing, see e.g. [Cazals and Pouget 2003].

Conjugate tangents play an important role here because they can be used to create a developable surface Ψ which is tangentially circumscribed to a given surface Φ along a curve s (see Figure 9). That *tangent developable* even has the nice property

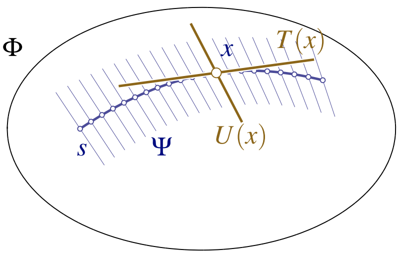


Figure 9: Consider a point x in a geodesic s which lies in the surface Φ. If $T(x)$ is tangent to the geodesic, compute $U(x)$ as being conjugate to $T(x)$. Then the union of all tangents $U(x)$ is a developable ruled surface Ψ which is tangentially circumscribed to Φ along the curve s.

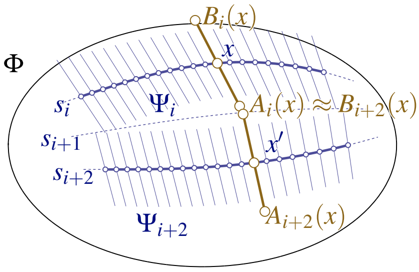


Figure 10: Developable surfaces Ψ_i associated with geodesics with *even* indices i are trimmed by geodesics with odd index.

that s is a geodesic not only for Φ, but also for Ψ. Thus, when Ψ is unfolded into the plane, s becomes a straight line.

This geometric information suggests the following algorithm to create panels: First, for all geodesics s_i in a given geodesic pattern compute the tangent developable Ψ_i according to Figure 9. Trim those surfaces along the intersection curves with their respective neighbours. Unfolding the trimmed Ψ_i's yields the flat state of panels.

Unfortunately this does not work in practice. One reason is that the rulings of the tangent developables may behave in weird ways. Another reason is that the intersection of neighbouring Ψ_s's is often ill-defined, so trimming as suggested will not work. We therefore have chosen the following modified procedure:

1. For the geodesics s_i where i is an even number compute the tangent developable Ψ_i according to Figure 9. That is, for a dense sample of points x on s_i we compute the rulings $U_i(x)$ which are conjugate to the tangent $T_i(x)$.

2. Delete all rulings $U_i(x)$ of Ψ_i where the angle enclosed with the tangent $T_i(x)$ is smaller than some threshold (say, 20 degrees) and fill the holes by interpolation (this is a standard procedure).

3. On each ruling $U_i(x)$ determine points $A_i(x)$ and $B_i(x)$ which are closest to the geodesics s_{i-1} and s_{i+1}, respectively (see Figure 10). This serves for trimming the surface Ψ_i.

4. Optimize globally the positions of points $A_i(x)$ and $B_i(x)$ such that trim curves are smooth, such that $A_i(x)$ and $B_i(x)$ are close to geodesics s_{i-1}, s_{i+1}, and such that the ruling segments $A_i(x)B_i(x)$ lie close to Φ. For this optimization we need the distance fields of Φ and of the single geodesics. We only change the surface a little bit and hope not to lose too much developability.

Figures 11, 12 and 13 illustrate panelizations of freeform shapes obtained by this method. The degree of developability which is achieved can be evaluated by measuring the Gauss curvatures of panel surfaces, such as done by Figure 16. The Gauss curvature vanishes for exact developability. The exact values for the panelizations of Figures 11 and 14, which work with the same design surface and comparable strip width can be seen in the table at the end of Section 4.

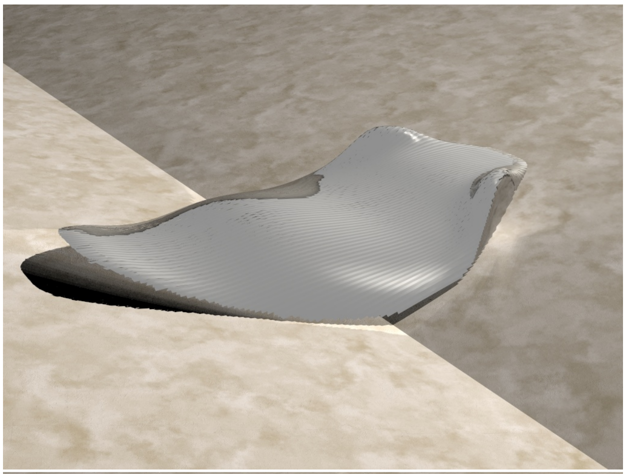


Figure 11: Almost-developable strips constituting a watertight surface.

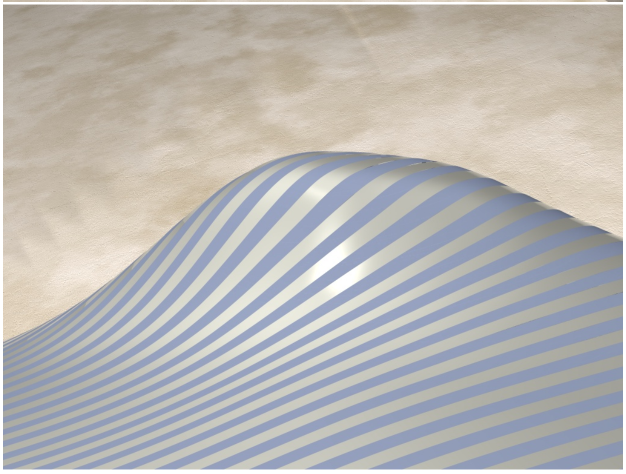


Figure 12: Detail of Figure 11. The gaps in between panels which occur in highly curved areas are hardly visible. The maximal strip width is 0.4% of the entire design's bounding box diagonal.

Figure 13: Watertight panels based on the segmentation and parallel transport methods. See also Figure 4. The intrinsic curvature of the rather broad panels is too high to make this design practicable: its purpose is to illustrate the parallel transport method.

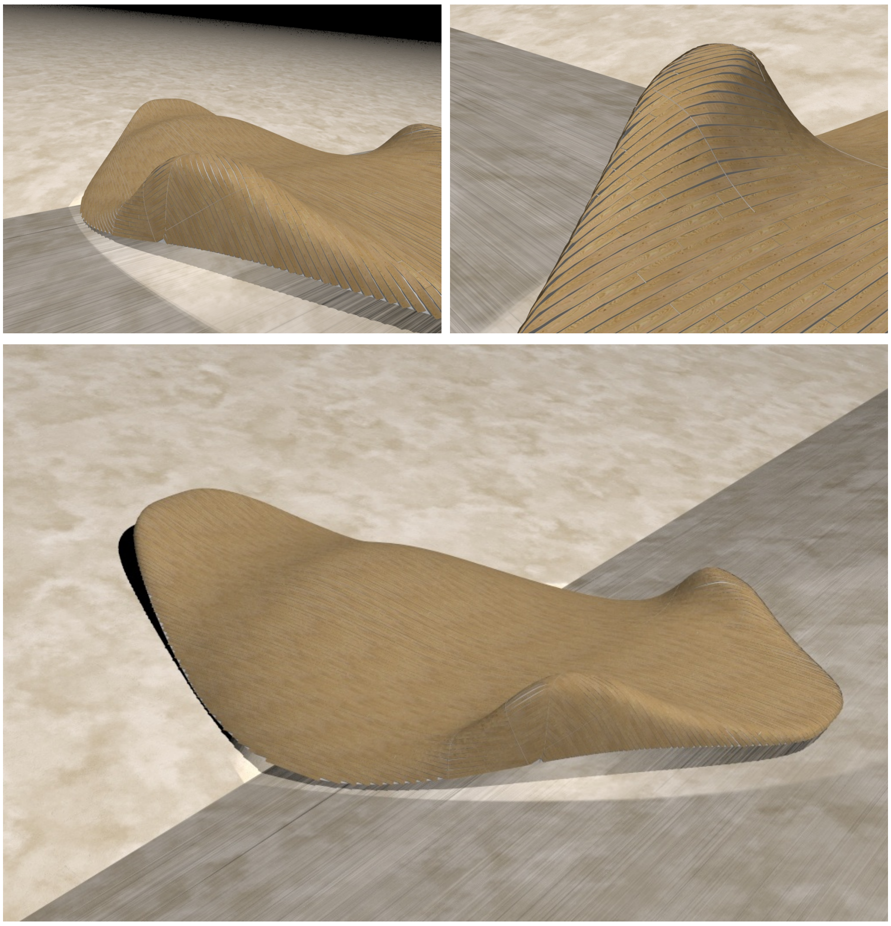


Figure 14: *Below:* A surface is covered by wooden panels of constant width. This is achieved by the 'evolution method' illustrated by Figure 5: The pattern of panels evolves from a well-placed source geodesic as long as the requirement of constant panel width is satisfied up to certain thresholds. If the panel width deviates too much from the desired value, the geodesics are broken. Subsequently panel surfaces have been created by the 'binormal method'. *Above:* Details. A further detail is shown by Figure 1.

3.2 The binormal method.

Our second method of defining panels (after a pattern of geodesics in the surface Φ has been found) works directly with the geodesic curves.

Assume that such a geodesic s is traversed by a point $P(t)$ moving with unit speed, where t is a time parameter. For each time t we have the velocity vector $T(t)$, the normal vector $N(t)$ of the surface Φ in the point $P(t)$, and a third vector $B(t)$ (the *binormal vector*) which makes T, N, B a moving orthogonal right-handed frame.

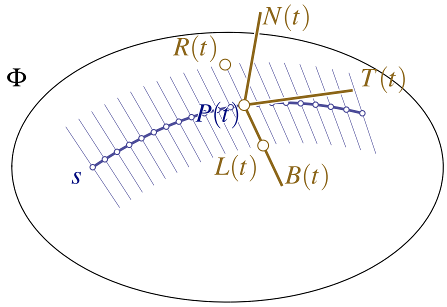


Figure 15: The binormal method defines a ruled panel surface from a central geodesic s via its Frenet frame T, N, B: The ruling passing through the central point $P(t)$ on the geodesic is indicated by the binormal vector $B(t)$. The endpoints of the ruling segment are points $L(t)$ and $R(t)$ whose distance from $P(t)$ is half the intended panel width.

For computational purposes, the surface Φ is represented as a triangle mesh and s is given as a polyline. Numerically the computation of the frame T, N, B is stable if performed in the way described above, despite the fact that it is actually the Frenet frame of s which usually exhibits numerical deficiencies (this connection with the Frenet frame follows from the geodesic property).

For each geodesic, the associated panel surface is constructed according to Figure 15. Panelizations of freeform surfaces which have been achieved with this method are shown by Figures 7 and 14.

3.3 Discussion

The previous two subsections proposed two different methods of defining ideal and mathematically abstract surfaces which are to be followed by panels. The 'tangent developable' method tries to produce panel surfaces which are achievable by pure bending (in fact the tangent developable is the only surface with this property which is also tangent to the original design surface). Thus the mathematical goal of developability is corresponding to a natural manufacturing goal. It seems reasonable to let actual panels exactly follow the surfaces proposed by this algorithmic method.

The situation is slightly different for the second suggested way of defining panel surfaces (the 'binormal' method). From a mathematical viewpoint it is a simple and obvious way of defining panel surfaces, but it is unclear that this surface should be the shape of a panel after it has been forced to follow a geodesic on the surface Φ. Of course such a shape is subject to the existing constraints, but one would assume that panels rather assume shapes achievable by pure bending. The purpose of the binormal method is mainly to pin down a mathematically exact surface, for the practical purpose of having shapes exactly defined. Anyway the following section shows that the panel shapes defined by the binormal method are admissible from the viewpoint of stresses and strain.

4 Stress and strain in panels.

This section investigates the deformation a rectangular strip of elastic material experiences when it is bent into the shape of a ruled surface Ψ such that the central line

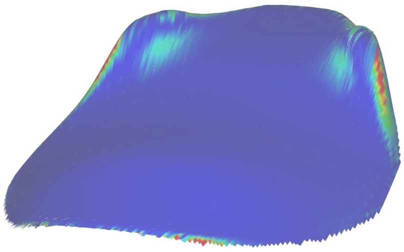


Figure 16: Visualization of Gaussian curvature of the design shown by Figures 11, 12. Blue corresponds to zero, red to the maximum value -0.02 (this means $\rho = 7.07$). The bounding box diagonal of this object is 188.

m of the strip follows a 'middle geodesic' s in Ψ. This applies to both our methods of defining panel surfaces. It seems a reasonable assumption that the central line is only bent, but not stretched. Due to the saddle shape (negative Gaussian curvature) of all ruled surfaces, the lines parallel to m at distance $d/2$ are not only bent, but also stretched. It is known that after introducing the *radius of Gaussian curvature* $\rho = 1/\sqrt{|K|}$, the relative increment in length (the strain) of the strip boundaries is given by

$$\varepsilon = \frac{1}{2}(d/2\rho)^2 + \cdots,$$

where the dots indicate terms of higher order in d. We are first concerned with tensile stress due to this stretching; for other stresses due to bending and shear see the end of this section. A rough estimate, expressing stress by $\sigma = E\varepsilon$, yields

$$d/2\rho \leq C, \quad \text{with} \quad C = \sqrt{2\sigma_{\max}/E},$$

where $\sigma_{\max}$ is the maximum admissible stress and E is Young's modulus. The approximative nature of our computation implies using a suitable safety factor when choosing $\sigma_{\max}$. The value C is a material constant which yields an upper bound $d_{\max} = 2\rho_{\min}C$ for the maximum strip with. With sample values for $\sigma_{\max}$ we get

material	Young modulus E $[N/mm^2]$	maximum stress (sample values) $\sigma_{\max}$ $[N/mm^2]$	constant $C = \sqrt{2\sigma_{\max}/E}$
steel	200000	250	0.05
wood	13000	80	0.11

Strip widths and their admissibility for models shown in this paper are collected in the following table. Since these examples have been selected mainly with a view towards visualization, some are not admissible. However they can easily be made so by choosing narrower panels. The choice of units in this table is arbitrary.

Figure No.	material	actual panel width $[m]$	$\lvert K\rvert_{\max}$ $[m^{-2}]$	$\rho_{\min}$ $[m]$	bounding box size $[m]$	admissible width $[m]$	admis-ible?
1, 5, 14	wood	$d = 0.7$	0.1	3.16	188	0.7	yes
4, 13	steel	$d \leq 0.1$	5	0.44	2.8	0.04	no
	wood					0.1	yes
11, 12	steel	$d \leq 0.8$	0.02	7.07	188	0.71	almost

Bending and shear stress. Both bending stress and shear stress for a panel with thin rectangular cross-section depend on the panel thickness h, but not on the panel width d if $h/d \ll 1$; the maximum values of these stresses (denoted by σ, τ in this paragraph) occur on the outer surface of the panel. These values depend on the curvature κ of the panel's central geodesic and the rate of torsion θ of the panel (we have $\sigma = E\kappa h/2$ and $\tau = hG\theta$, where G is the shear modulus). Clearly the panel surfaces obtained by the 'tangent developable' method experience less shear than the ones created by the 'binormal' method. It is a standard matter to combine all stresses (tension, shear, bending) and use this information for checking if the panel's dimensions are admissible.

It is interesting to know how the rate of torsion θ (twist angle per panel length) is related to the Gaussian curvature of the panel: It is known that θ, measured in arc per meter, does not exceed $\sqrt{|K|} = 1/\rho$, where the maximum value occurs in case the central geodesic's tangent happens to be an asymptotic direction of the panel surface [do Carmo 1976].

5 Conclusion.

This paper treats paneling of freeform surfaces with rectangular (or almost-rectangular) panels, which are known to follow geodesic curves. For the layout of a system of geodesics several methods have recently been published. We survey some of them in this paper, especially those which produce geodesics running approximately parallel to each other. We further discuss the panel surfaces themselves under the viewpoint of panel shapes achievable by pure bending and a watertight overall panel surface, and we demonstrate our methods by means of some examples. Finally we discuss tensile and shear stresses in panels which occur when they are mounted on freeform surfaces.

Future research. The connection between geometry and mechanics is a very important and at the same time most challenging issue in any freeform design. One topic of future research therefore is to combine geometric considerations with simple aspects of mechanics – our way of expressing stresses by Gaussian curvature already points in this direction.

Panelization poses many geometric questions whose systematic investigation would be rewarding: For instance, panels in the shape of generalized cylinders which are important for bent glass; and more generally special shapes of panels which are relevant for certain manufacturing techniques and specific applications in building construction. Our aim must generally be to find *construction-aware design tools* which do not generate shapes first and lets us think about manufacturing afterwards, but tools which actively, during the design phase, incorporate the side conditions engendered by manufacturing constraints.

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