Viscous Resuspension of a sediment caused by oscillating stratified flows

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Abstract

It is known that a sediment of settled particles can resuspend under the action of shear. In this paper, based on a mathematical model given by LEIGHTON & ACRIVOS [6], we study theoretically the viscous resuspension of a sediment in a Couette channel with harmonically oscillating walls. Numerical experiments reveal that the resuspension height and the particle volume concentration at the bottom of the channel depend on the frequency of the oscillation. While oscillation of the top wall has nearly no influence on the sediment, a moving bottom wall causes the settled particles to completely resuspend if the frequency is large enough.

1 Introduction

A settled bed of negatively buoyant particles can resuspend when in comes in contact with a clear fluid, even if Brownian motion or turbulence at very small Reynolds numbers are absent. This phenomenon, called *viscous resuspension*, was first observed by GADALA-MARIA [3] and GADALA-MARIA & ACRIVOS [4], and has been the subject of several experimental and theoretical investigations. LEIGHTON [6] and SCHAFLINGER et al. [9] have studied both theoretically and experimentally viscous resuspension caused by a laminar stratified flow. Theoretical predictions for a Couette gap and a 2-dimensional Hagen-Poiseuille channel are in good agreement with the experimental results.

Even though small frequency oscillatory shear experiments performed by GADALA-MARIA & ACRIVOS [4] indicate anisotropic structure of the suspension, we model the two phase mixture as an effective Newtonian fluid. This is justified for larger frequencies, because the particles do not have enough time to rearrange. ACRIVOS [1] conducted a survey on viscous resuspension and the rheology of concentrated suspensions of con-colloidal particles.

In the present paper we study viscous resuspension of a sediment in a Couette channel with a harmonically oscillating top or bottom wall. The theory is based on a mathematical model for hydrodynamic diffusion introduced by LEIGHTON & ACRIVOS [6]. This model later was successfully employed by ACRIVOS et al. [2], SCHAFLINGER et a. [9], and ZHANG & ACRIVOS [10].

The downward flux of particles is governed by Stokes' law and the upward hydrodynamic lift is governed by a diffusion coefficient proportional to the shear rate and the square of the particle radius. The Navier-Stokes equations and the hydrodynamic diffusion equation then have to be solved simultaneously. Besides the relative density difference, the amount of total sediment in the channel and the frequency, the characteristic parameters of the problem are a Reynolds number and a kind of Shields number.

2 Basics

We consider a suspension of negatively buoyant particles in a Couette channel, as depicted in figure 1. We assume that the flow is periodic and unidirectional. The symbol h_c denotes the height of a remaining sediment layer at the bottom and h is the z-coordinate of the top of the resuspended layer in presence of the laminar shear flow with velocity $u = u(z, t)$. Both h_c and h depend on the time t. The total height of the channel is given by h_p, and the height of the sediment layer at the bottom, if the flow were suddenly turned off is denoted by h_0. The symbols μ and ρ denote effective viscosity and density, respectively. When necessary, subscript 1 indicates physical properties of the clear fluid, subscript 2 physical properties of the particles, and no subscript indicates physical properties of the mixture. The particle volume concentration is denoted by ϕ, and ϕ_0

denotes the particle volume concentration of the sediment. For our purpose we set $\phi_0 = 0.58$. The letter g denotes the gravitational acceleration.

For simplicity, we restrict ourselves to the resuspension of spheres of equal radius a and density ρ_2. Then, under presence of gravity the downward flux N_s of particles is given by

$$N_s = \phi u_0 f(\phi), \tag{1}$$

where u_0 is the sedimentation velocity of a single particle in the clear fluid according to Stokes' law:

$$u_0 = \frac{2}{9}\frac{a^2 g(\rho_2 - \rho_1)}{\mu_1}. \tag{2}$$

Here $f(\phi)$ is the hindrance function of a single particle settling in the presence of other particles in the suspension, i. e., the ratio of the sedimentation velocity of a single sphere in the suspension to Stokes' settling velocity. The hindrance function $f(\phi)$ can be easily estimated by the settling of a single sphere in a suspension with effective viscosity and density. Thus,

$$f = \frac{1-\phi}{\mu_r}, \tag{3}$$

where μ_r is the dimensionless effective viscosity μ/μ_1 of the suspension. The empirical relationship

$$\mu_r = \frac{\mu}{\mu_1} = \left(1 + \frac{1.5\phi}{1 - \phi/\phi_0}\right)^2 \tag{4}$$

was found by LEIGHTON [5] in order to represent experimental data.

It is known that an "effective viscosity" derived from experimental data can vary in different experiments as much as an order of magnitude as the particle concentration reaches its maximum [1]. However, since nothing is known for the effective viscosity of concentrated suspensions undergoing oscillatory shear, we assume eq. (4) to be valid.

The considerations above require the particle Reynolds number to be small, i. e., $Re_p \ll 1$. If the size of the particles is large enough, Brownian motion does not have a significant influence and non-hydrodynamic forces are negligible. In this case, sedimentation in the direction of gravity is counterbalanced by a hydrodynamic lift due to particle-particle interactions. The upward diffusive flux from regions of high concentration to low is given by

$$N_d = -D\frac{d\phi}{dz}, \tag{5}$$

with D being the shear-induced diffusion coefficient. According to [5] it can be approximated by

$$D = \hat{D}\dot{\gamma}a^2 = \frac{1}{3}\phi^2(1 + \frac{1}{2}e^{8.8\phi})\dot{\gamma}a^2, \tag{6}$$

with $\dot{\gamma}$ being the absolute value of the shear rate. If τ denotes the shear stress, the shear rate is given by

$$\dot{\gamma} = \left|\frac{\tau}{\mu}\right|. \tag{7}$$

Acrivos [1] described a shear-induced anisotropy that occurred in oscillatory experiments, which show that in the absence of non-hydrodynamic forces, suspensions of monodisperse solid spheres do not behave as effective fluids with a concentration dependent effective viscosity. He [1] supposed that the, evidently non-Newtonian, experimental results can be explained by the fact that if the direction of the shear is suddenly reversed, the particles do not immediately rearrange themselves into a mirror image of their original configuration. Thus, the oscillatory shear experiment described by Acrivos [1] led to a non-harmonic oscillation in the shear stress which became asymptotically harmonic if the amplitude of oscillation tended to zero.

On the other side, relatively large frequencies do not allow the particles to rearrange and thus we assume that the suspension is again isotropic. Certainly, non-Newtonian effects deserve further study, which is, however, beyond the scope of this paper. In the following we will neglect any anisotropies or non-Newtonian effects.

3 Oscillatory Couette flow

If v denotes the average particle velocity along the direction of gravity, and j_2 the particle flux along this direction, then $j_2 = \phi \cdot v$ and the equation of particle mass balance reads

$$\frac{\partial \phi}{\partial t} + \frac{\partial j_2}{\partial z} = 0. \tag{8}$$

In our case

$$j_2 = N_s - N_d = \phi u_0 f(\phi) - \hat{D}(\phi)\dot{\gamma} a^2 \frac{\partial \phi}{\partial z}. \tag{9}$$

For a Newtonian fluid the Navier-Stokes equations reduce to

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}(\mu \frac{\partial u}{\partial z}). \tag{10}$$

Here we would like to note that we assume that there is no relative motion between the particles and the continuous fluid in x-direction. We introduce a set of dimensionless parameters which completely characterize our problem. We scale lengths along the direction of the z-axis with h_p, time with h_p/u_0 and lengths along direction of the x-axis with $u_w h_p/u_0$, where u_w is a characteristic wall velocity. Then the velocity u in x-direction will scale with u_w. For u_w we take the velocity amplitude of the oscillation. With a Reynolds-number

$$Re = \frac{\rho_1 u_0 h_p}{\mu_1}, \tag{11}$$

a relative density difference

$$\varepsilon = \frac{\rho_2 - \rho_1}{\rho_1}, \tag{12}$$

and a type of Shields number

$$\kappa = \frac{u_w}{u_0}\left(\frac{a}{h_p}\right)^2 = \frac{9 u_w \mu_1}{2 h_p^2 g(\rho_2 - \rho_1)}, \tag{13}$$

the dimensionless versions of eqs. (8) and (10) read

$$Re(1 + \phi\varepsilon)\frac{\partial u}{\partial t} - \frac{\partial}{\partial z}(\mu_r \frac{\partial u}{\partial z}) = 0, \tag{14}$$

and

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}\left(\phi f(\phi) + \kappa \hat{D}(\phi)\left|\frac{\partial u}{\partial z}\right|\frac{\partial \phi}{\partial z}\right) = 0. \tag{15}$$

Besides the frequency, the set of dimensionless parameters describing the problem consists of the four numbers κ, Re, ε and the amount of particles present in the suspension, which can be expressed by the dimensionless height h_0 of the sediment if the oscillation were suddenly turned off.

We see that the particle radius, which is contained in the Reynolds number but not in κ, only appears in the first term of eq. (14). Therefore, a steady state resuspension flow does not depend on the particle radius. This surprising fact has already been mentioned by LEIGHTON & ACRIVOS [6].

4 Results

Analytical solutions of eqs. (14) and (15) are possible in the case of a steady state resuspension Couette flow [6] and in the case of a harmonically oscillating clear fluid. Then, the Navier-Stokes equations simplify to

$$Re\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2}. \tag{16}$$

In dimensionless form, the boundary conditions are

$$\begin{aligned} u(0,t) &= \cos\omega t, \\ u(1,t) &= u_1\cos(\omega t + \psi). \end{aligned} \tag{17}$$

Then the velocity u is

$$\begin{aligned} u(z,t) &= R_1 e^{\alpha z}\cos(\psi_1 + \alpha z + \omega t) \\ &+ R_2^{-\alpha z}\cos(\psi_2 - \alpha z + \omega t). \end{aligned} \tag{18}$$

Equation (18) contains the constants R_1, R_2, ψ_1, ψ_2 which follow from the boundary conditions (17), and the parameter α which is defined as

$$\alpha^2 = \frac{1}{2}Re \cdot \omega. \tag{19}$$

The general case of a resuspension flow with oscillating walls, however, seems analytically intractable.

Because we do not want to solve the equation for the most general boundary conditions, but only for periodic ones, we can use a simple second order finite difference scheme which, on the one hand, is stable against stiffness of the equations which will inevitably occur when the concentration approaches ϕ_0, and on the other hand, the quantity of error can be made arbitrarily small uniformly by refining the discretization.

In the case of a steady state Couette resuspension flow (cf. [6]) it does not matter whether the top or the bottom wall is moving. In the oscillating case, however, the situation is different. If we start with a sediment of certain height h_0, and the top wall begins to oscillate, it is possible that no resuspension occurs at all, because the average velocity in the clear fluid above the sediment decreases with $e^{-\alpha z}$, as can be seen from eq. (18).

If, on the other hand, the bottom wall is oscillating, the situation is different. We found that the particles will even completely resuspend if the frequency of the oscillation is large enough. The resuspended layer will fill the channel up to a height h which, in general, depends on time t. However, if

$$u_0 \ll \omega h_p, \tag{20}$$

oscillation of the resuspension height h can be neglected. The inequality (20) can be derived from the characteristic time for settling h_p/u_0, if the resuspension height h is of the same magnitude as the channel height h_p, and the characteristic time for oscillation $1/\omega$.

Examples of velocity profiles for different times and of a concentration profile for the case of an oscillating bottom wall are shown in figures 2a and 2b. The particle volume concentration at the bottom wall ($z = 0$) is less than ϕ_0, which shows that in this case resuspension is complete (fig. 2a). Condition (20) is fulfilled and the concentration profile does not visibly depend on time. Just below $z = h$ the particle volume concentration and thus the effective viscosity change rapidly. This implies that the effective viscosity decreases rapidly if z increases. The effect of the change in viscosity is also clearly visible in the velocity profiles (fig. 2b).

Figure 3a shows the resuspension height h dependent on the dimensionless angular velocity ω for some fixed values of the Reynolds number Re, the Shields parameter κ and the initial sediment height h_0. We can see that for some angular velocity $\omega^\star$, which depends on h_0, on κ and on Re, the resuspension height is maximal. It is clear that h has to tend to zero if ω tends to infinity, because then the suspension will not be able to follow the bottom wall. Also it has to be noted that for small ω the resuspension height will visibly depend on time and a plot would make no sense in this region.

Figure 3b shows the particle volume concentration at the bottom wall for the same parameter values. It can be seen that for $\omega > \omega_0$ the particle concentration is less than the particle concentration in a sediment and for $\omega > \omega_0$ *complete* resuspension occurs.

5 Conclusions

In this paper we studied theoretically and numerically viscous resuspension of a sediment in a two-dimensional Couette channel undergoing oscillatory shear. The theoretical approach was based on a mathematical model in which the net downward flux of particles due to gravity is counterbalanced by a diffusive flux caused by a shear-induced random motion of the particles [5]. This model does not include Brownian motion which is negligible within the range of particle diameters investigated. We also neglect any non-Newtonian effects. The resulting differential equations depend on a Reynolds number, a Shields type parameter and a relative density difference between the particles and the fluid. The boundary conditions depend on the oscillation of the walls and the initial conditions on the total amount of sediment. These equations had to be solved numerically. The results showed that an oscillating top wall does not influence the sediment, but an oscillating bottom wall causes the settled particles to completely resuspend if the frequency is large enough.

The results of this work show once more how viscous effects can lead to resuspension even when turbulence is completely absent at very small Reynolds numbers. Although in most industrial processes involving sediments and suspensions, e. g. the transport of sediments, the flow is highly turbulent and viscous resuspension is negligible, there are also situations where this phenomenon becomes important.

Topics of further research in this direction should include a comparison of numerical with experimental results, which would also provide more data concerning the diffusion coefficient which describes the diffusive upward flux of particles. At the same time, it would be interesting to have a satisfactory theoretical explanation of non-Newtonian effects which are clearly visible in some situations, as described by ACRIVOS [1].

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Figure captions:

Figure 1: Couette gap with suspension

Figure 2: Concentration profile (a) and velocity profiles (b). $Re = 21.8$, $\kappa = 0.046$, $h_0 = 0.605$, $\varepsilon = 0.1$, $\omega = 22.9$.

Figure 3: Resuspension height (a) and particle volume concentration at bottom (b) dependent on the dimensionless angular velocity ω. Fixed parameters are $Re = 21.8$, $\varepsilon = 0.1$, $\kappa = 0.046$. Bottom wall velocity is oscillating in the interval $-1, 1$.