

MATH 1300 A, Fall 2013
Solution Quiz 3

1. (100 points) Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$ for

$$f(x) = \sqrt{\frac{\sqrt{x} + 11}{x^3 + 2}}$$

The equation of the tangent line at $x = x_0$ is

$$y = f(x_0) + f'(x_0)(x - x_0)$$

At $x = 1$ we obtain

$$f(1) = \sqrt{\frac{\sqrt{1} + 11}{1^3 + 2}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

The derivative of f is given by

$$f'(x) = \frac{1}{2\sqrt{\frac{\sqrt{x}+11}{x^3+2}}} \cdot \frac{\frac{1}{2\sqrt{x}}(x^3+2) - 3x^2(\sqrt{x}+11)}{(x^3+2)^2}$$

We substitute $x = 1$ and get

$$\begin{aligned} f'(1) &= \frac{1}{2\sqrt{\frac{\sqrt{1}+11}{1^3+2}}} \cdot \frac{\frac{1}{2\sqrt{1}}(1^3+2) - 3 \cdot 1^2(\sqrt{1}+11)}{(1^3+2)^2} \\ &= \frac{1}{4} \cdot \frac{\frac{3}{2} - 36}{9} = -\frac{69}{72} \\ &= -\frac{23}{24} \end{aligned}$$

Replacing the values of $f(1)$ and $f'(1)$, we obtain that the equation of the tangent line is

$$y = 2 - \frac{23}{24}(x - 1) = \frac{71}{24} - \frac{23}{24}x$$