1. (*50 points*) Find the derivative $\frac{dy}{dx}$ for the implicitly defined function

$$1 + \cot(x^2 + 1) = \arctan(y^2)$$

Taking the derivative with respect to x in both sides of the equation we get

$$-\csc^2(x^2 + 1) \cdot 2x = \frac{1}{1 + (y^2)^2} 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-\csc^2(x^2 + 1) \cdot x \cdot (1 + y^4)}{y}$$

2. (*50 points*) Determine the intervals on which the following function is increasing and the intervals on which it is decreasing. Find its critical points and identify them as local minima, local maxima, or non local extrema.

$$f(x) = x^3 - 3x^2 + 3x + 8$$

The derivative of f is

$$f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1) = 3(x - 1)^2$$

Analyzing the sign of f' we get

$$f' : + + + + + (x = 1) + + + + + +$$
$$\text{increasing} \qquad \text{increasing}$$

Thus, f is increasing everywhere. It has a critical point at $x = 1$, which is a non-local extremum.