MATH 1300 A, Fall 2013
Solution Quiz 5

1. (*50 points*) Find the limit

$$\lim_{x\to 0} \frac{x-\sin x}{x\cos x - x}$$

Substitution of $x=0$ in the above fraction produces $\frac{0-0}{0-0}=\frac{0}{0}$, so we can apply L'Hopitals Rule:

$$\lim_{x\to 0} \frac{x-\sin x}{x\cos x - x} = \lim_{x\to 0} \frac{1-\cos x}{\cos x - x\sin x - 1}$$

Substitution of $x=0$ produces $\frac{1-1}{1-0-1}=\frac{0}{0}$ again, so we can apply L'Hopitals Rule a second time:

$$\lim_{x\to 0} \frac{x-\sin x}{x\cos x - x} = \lim_{x\to 0} \frac{\sin x}{-\sin x - \sin x - x\cos x} = \lim_{x\to 0} \frac{\sin x}{-2\sin x - x\cos x}$$

Substitution of $x=0$ produces $\frac{0}{-0-0}=\frac{0}{0}$ again, so we can apply L'Hopitals Rule a third time:

$$\lim_{x\to 0} \frac{x-\sin x}{x\cos x - x} = \lim_{x\to 0} \frac{\cos x}{-2\cos x - \cos x + x\sin x} = \frac{1}{-2-1+0} = -\frac{1}{3}$$

2. (*50 points*) A soccer ball is kicked directly upwards from the ground. If the ball rises 80 feet in the first second, how high will the ball go?

The equations for the position and velocity of the ball are

$$s(t) = -16t^2 + v(0)t + s(0) \qquad\qquad v(t) = -32t + v(0)$$

Since the initial position of the ball is $s(0)=0$ and $s(1)=80$ we obtain

$$80 = s(1) = -16 + v(0)$$

$$96 = v(0)$$

In the highest point the ball has velocity equal to zero:

$$v(t) = -32t + 96 = 0$$

which happens when $t=3$ seconds. Therefore the highest position of the ball is attained at $t=3$ and is equal to

$$s(3) = -16(3)^2 + 96(3) + 0 = 144 \text{ feet}$$