Exercise 1

Use Haiman's operator theorem to:

(i) Compute DH_3 explicitly.

(ii) Show that $\dim(DH_3) = 4^2 = 16$ (number of parking functions).

(iii) Show that

$$\mathrm{Hilb}_{DH_3}(q,t) = 1 + 2q + 2t + 2q^2 + 3qt + 2t^2 + q^3 + q^2t + qt^2 + t^3.$$

t^3	1			
t^2	2	1		
t	2	3	1	
1	1	2	2	1
	1	q	q^2	q^3

Note that this polynomial is symmetric in q and t.

Exercise 2

In the previous exercise ($n = 3$), verify that:

(i) $\mathrm{Hilb}_{DH_n}(q,0) = [n]_q!$

(ii) $q^{\binom{n}{2}} \mathrm{Hilb}_{DH_n}(q,q^{-1}) = [n+1]_q^{n-1}$

(iii) $\mathrm{Hilb}_{DH_n}(q,1) = \sum_{P\in\mathrm{Park}(n)} q^{\mathrm{area}(P)}$

Exercise 3

Let Alt_n be the subspace of alternants of DH_n and define the q,t-Catalan polynomial as $c_n(q,t) := \mathrm{Hilb}_{\mathrm{Alt}_n}(q,t)$.

(i) Compute Alt_3 explicitly.

(ii) Show that

$$c_3(q,t) = q^3 + q^2t + qt + qt^2 + t^3.$$

Note that this polynomial is symmetric in q and t.

Exercise 4

In the previous exercise ($n = 3$), verify that:

(i) $q^{\binom{n}{2}} \mathrm{Hilb}_{\mathrm{Alt}_n}(q,q^{-1}) = \frac{1}{[n+1]_q} \begin{bmatrix} 2n \\ n \end{bmatrix}_q$

(ii) $\mathrm{Hilb}_{\mathrm{Alt}_n}(q,0) = \sum_{\pi\in\mathrm{Dyck}(n)} q^{\mathrm{area}(\pi)}$.