Cesar Ceballos Institute of Geometry TU Graz

# Discrete and Computational Geometry Exercise Sheet 02

Winter Semester 2024-2025

Bonus exercises are not mandatory but can be presented for bonus points. In Exercise 5, only part (i) is "mandatory" and can be marked as solved for the exercise session.

### Exercise 1

For each n > 3, find a polygon with n vertices that has a unique triangulation.

#### Exercise 2

Find the number of distinct triangulations of each of the following polygons:













### **Exercise 3**

A polyhedron is a 3D version of a polygon. A tetrahedralization of a polyhedron is a partition of its interior into tetrahedra whose edges are diagonals of the polyhedron.

- (i) find two different tetrahedralizations of a 3-dimensional cube.
- (ii) do all tetrahedralizations of the 3-dimensional cube have the same number of tetrahedra? justify your answer.

## **Exercise 4**

Show that the n-dimensional cube can be triangulated into exactly n! simplices.

### **Exercise 5**

The Fortress Problem: How many guards are needed to see the exterior of a polygon P if the guards can be placed at its vertices? Here, an exterior point y is seen by a guard at vertex x if the segment xy does not intersect the interior of the polygon.

- (i) show that  $\lceil \frac{n}{2} \rceil$  guards are necessary for some polygons.
- (ii) (Bonus exercise) find an example where neither placing guards at even nor at odd vertices leave the exterior of the polygon uncovered.
- (iii) (Bonus exercise) show that  $\lceil \frac{n}{2} \rceil$  guards are sufficient to see the exterior of a polygon with n vertices.