Exercise 1

Show that a diagonal between any two nonadjacent vertices of a polygon P exists if and only if P is a convex polygon.

Exercise 2

Find bijections between triangulations of a convex polygon with $n+2$ vertices and:

(i) binary trees with n nodes.

Here, a binary tree is a rooted tree in which each node has at most two children, referred to as the left child and the right child.

(ii) Dyck paths inside an $n \times n$ square.

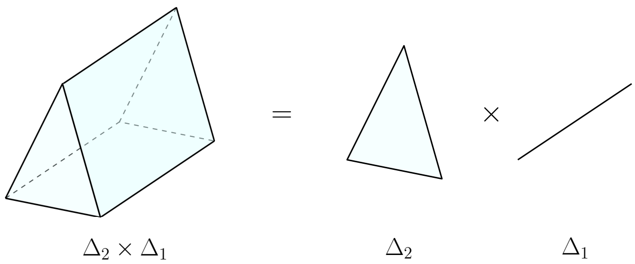
Here, a Dyck path is a lattice walk on the plane from the point $(0,0)$ to (n,n), using unit North and East steps, that stays above the diagonal $y = x$.

Exercise 3

Let P, Q, R be three polygons on the plane. Show that if P is scissors congruent to Q, and Q is scissors congruent to R, then P is scissors congruent to R. In other words, show that scissors congruence is transitive.

Exercise 4

(i) How many tetrahedralizations does a prism over a triangle have?



(ii) (Bonus exercise) Find $n!$ different triangulations of $\Delta_{n-1} \times \Delta_1$.

Here, the n-dimensional simplex Δ_n is defined by

$$\Delta_n = \operatorname{conv}\{e_1, e_2, \ldots, e_{n+1}\} \subseteq \mathbb{R}^{n+1},$$

where e_i denote the standard basis vectors. The product $\Delta_m \times \Delta_n$ of two simplices is the set of points

$$\Delta_m \times \Delta_n = \{(p,q) : p \in \Delta_m \text{ and } q \in \Delta_n\} \subseteq \mathbb{R}^{(m+1)+(n+1)},$$

and has dimension $m+n$. For instance, $\Delta_2 \times \Delta_1$ is a prism over a triangle as illustrated above. Note that even though it lies in $\mathbb{R}^5$, it can be drawn as a 3-dimensional figure.

(iii) (Bonus exercise) Show that $\Delta_{n-1} \times \Delta_1$ has exactly $n!$ many different triangulations.