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Discrete and Computational Geometry

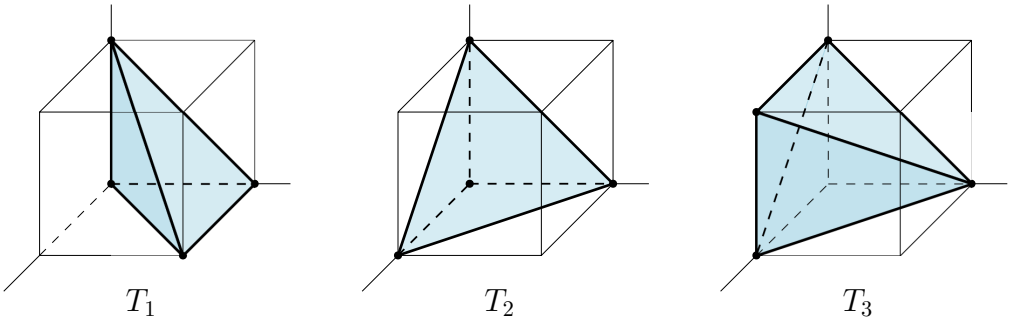
Exercise Sheet 04

Winter Semester
2024-2025

Exercise 1

(i) Find the dihedral angles of a regular tetrahedron.

(ii) Find the dihedral angles of the following tetrahedra embedded in a unit cube:



T_1 T_2 T_3

Exercise 2

For the tetrahedra in the previous exercise:

(i) show that T_1 is not scissors congruent to neither T_2 nor T_3

(ii) Are T_2 and T_3 scissors congruent?

(iii) show that T_1 is scissors congruent to a cube of the same volume

(iv) (bonus) find a dissection of T_1 and reassemble the pieces to form a cube

Exercise 3

Let $P \subseteq \mathbb{R}^d$ be a finite collection of points. A point $p \in P$ is called a *vertex* of $\mathrm{conv}(P)$ if

$$p \notin \mathrm{conv}(P \setminus \{p\}).$$

Let $V \subseteq P$ be the set of vertices of P. Show that $\mathrm{conv}(V) = \mathrm{conv}(P)$.

Exercise 4

Let P be a finite collection of points on the plane not lying on a single line. Show that:

(i) $\mathrm{conv}(P)$ is the convex polygon with the smallest perimeter containing P.

(ii) $\mathrm{conv}(P)$ is the convex polygon with the smallest area containing P.