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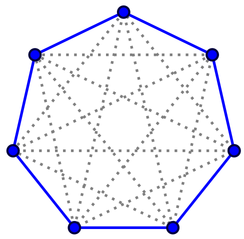
Exercise 1

Tverberg's Theorem implies that a set P of 7 points in the plane can be partitioned into three sets P_1, P_2, P_3 such that

$$\bigcap_{i=1}^{3} \operatorname{conv}(P_i) \neq \emptyset.$$

A point in the intersection of such a partition is called a Tverberg point of P. Let P be the set of vertices of a regular heptagon. How many Tverberg's point does P have? draw them in the figure below.



Exercise 2

(i) Show that any two compact convex sets $C, D \subseteq \mathbb{R}^d$ with $C \cap D = \emptyset$ can be strictly separated by a hyperplane h, i.e. C and D are on opposite open half-spaces with respect to h.

(ii) Find an example of two disjoint, non-empty, closed convex sets that can not be strictly separated.

Exercise 3

Let $A_1, \dots, A_{d+1}$ be sets in $\mathbb{R}^d$ such that a point $q \in \operatorname{conv}(A_i)$ for every i. Show that it is possible to find $a_i \in A_i$ such that $q \in \operatorname{conv}\{a_1, \dots, a_{d+1}\}$.

Note: if all A_i's are equal, this reduces to Caratheodory's Theorem.

Exercise 4

For Graham's Scan convex hull algorithm one needs to test whether a point r lies left or right of the directed line through two points p and q. Let $p = (p_x, p_y)$, $q = (q_x, q_y)$ and $r = (r_x, r_y)$.

(i) Show that the sign of the determinant

$$D = \begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines whether r lies left or right of the line.

(ii) Show that $|D|$ is twice the area of the triangle determined by p, q, and r.