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Discrete and Computational Geometry
Exercise Sheet 10

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Exercise 1

Consider the duality between points and non-vertical lines in the plane defined as follows:

- for a point $p = (a, b) \in \mathbb{R}^2$ its dual line is $D(p) = p^* : \; y = ax - b$,
- for a line $\ell : \; y = mx + n$ its dual point is $D(\ell) = \ell^* = (m, -n)$.

Show that D is order preserving, meaning that a point p is above the line ℓ if and only if the point ℓ^* is above the line p^*.

Exercise 2

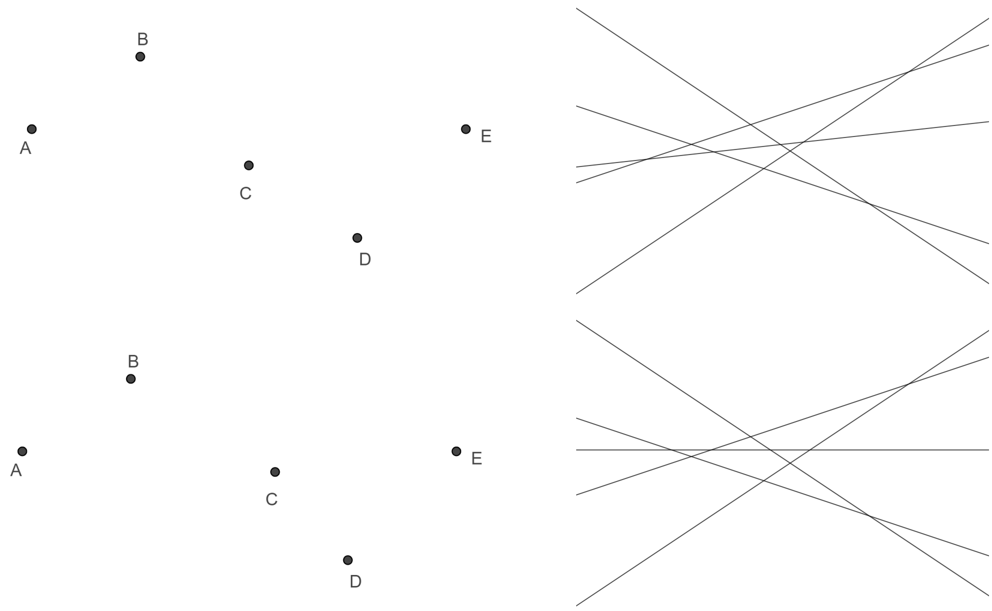
Let D be the map from previous exercise.

(i) Show that if p is a point on the parabola $y = x^2/2$ then $D(p) = p^*$ is the tangent line to the parabola at p.

(ii) For a point q, let p be its vertical projection to the parabola. Show that q^* is the line parallel to p^* at a vertical distance opposite to the distance from p to q.

Exercise 3

The following figure shows two point sets and two line arrangements. Which line arrangement corresponds to which point set?



Exercise 4

Consider a finite set $P \subset \mathbb{R}^2$ of points in general position (no four points on a circle), and let $\mathrm{Del}(P)$ be its Delaunay triangulation. Let q be a point inside the convex hull of P, such that $P \cup \{q\}$ is in general position. We say that a triangle t of $\mathrm{Del}(P)$ is *marked* if its circumcircle contains q.

(i) Show that the union of the marked triangles is a triangulated polygon Q containing q.

(ii) Show that the edges connecting p with the vertices of this triangulated polygon Q are inside Q.

(iii) Show that $\mathrm{Del}(P \cup \{q\})$ is the triangulation obtained from $\mathrm{Del}(P)$ by discarding the diagonals of the polygon Q, and replacing them by the edges from q to each of its vertices.