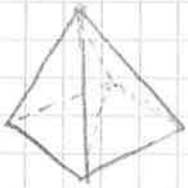
Discrete and Computational Geometry
Cesar Ceballos

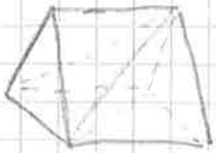
Triangulations in 3D

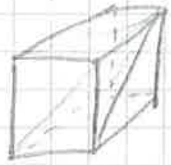
Last time : - Triangulations in 2D
Today : - Triangulations in 3D/higher dim
- Art Gallery in 3D
- Scissors Congruence

Tetrahedron : - Analog of a triangle in 3D.
- pyramid with a triangular base



Can we decompose more complicated 3D objects/ polyhedra into simpler pieces / tetrahedra?





A polyhedron P is the 3D version of a polygon, a 3D solid bounded by finitely many polygons.

A tetrahedralization of a polyhedron is a partition of its interior into tetrahedra whose edges are diagonals of the polyhedron (i.e. connect vertices of it)

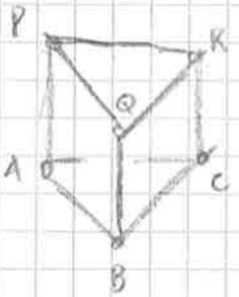
Some natural questions :

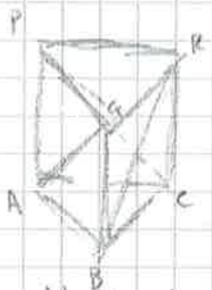
(1) How many tetrahedra does a tetrahedralization of P have? Is this number independent of the tetrahedralization? NO!

(2) Can every polyhedron be tetrahedralized? NO!

(3) Does every polyhedron have a diagonal in its interior? NO!

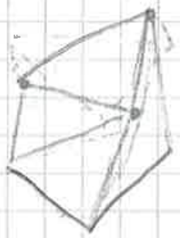
The Schönhardt polyhedron gives a negative answer to (2) and (3)

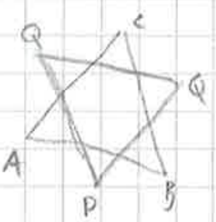




add three diagonals
AQ, BR, CP

rotate top
↳ triangle





rotate triangle PQR ↳
and keep all other triangles
Polyhedron is the solid bounded by all these triangles

→ No diagonal
→ Can not be tetrahedralized!

A negative answer to (1) is given by the 3D cube.

Exercise: Find two triangulations of the cube with 5 and 6 tetrahedra respectively

In <u>higher dimensions</u> things get even worst / more interesting!

A <u>simplex</u> : generalization of the notion of triangle or tetrahedron.

simplex  | convex hull of $n+1$ points that are affinely independent.

dimension	0	1	2	3	----	n

A <u>triangulation</u> is a decomposition of an n-dimensional object (Polytope) into simplexes with vertices being vertices of the polytope.

<u>Open Problem</u> What is the minimum number of simplices that a triangulation of the n-dimensional cube can have?

$n=3 \to 5$ Unknown! except for small values of n

$=4 \to 16$

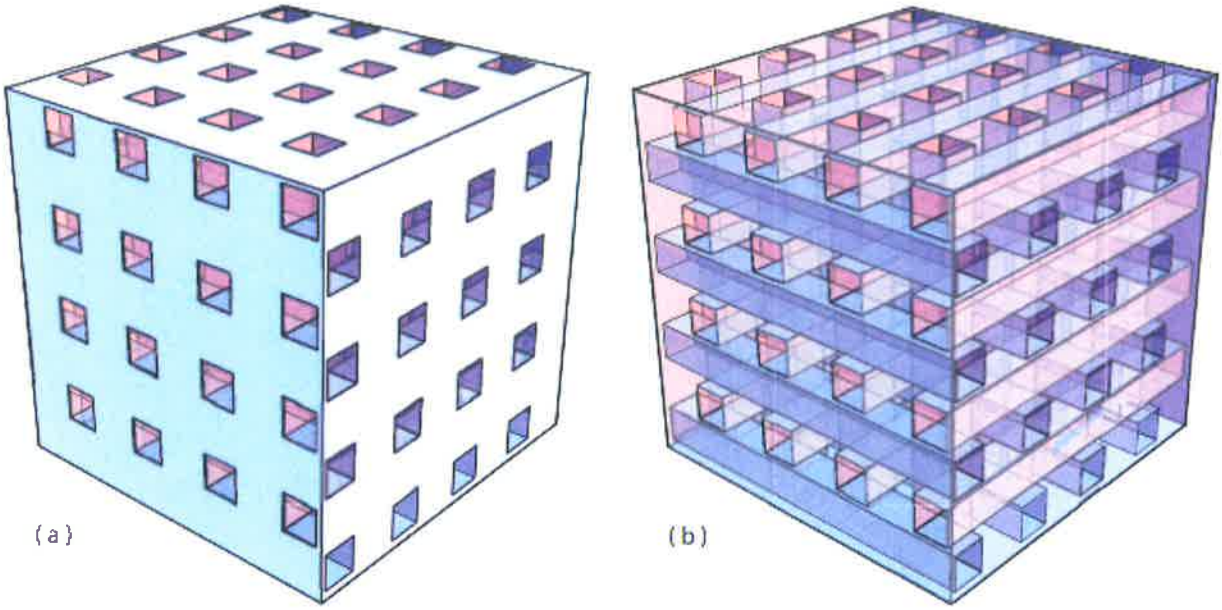


Figure 1.17. (a) The Seidel polyhedron with (b) three faces removed to reveal the interior. Devadoss and O'Rourke. Discrete and Computational Geometry.

Art Gallery Problem in 3D

The seidel polyhedron shows an example of a gallery that can not be enterily observed by placing guards in all the vertices of the polyhedron !

Question. Are any two polygons of the same area scissors congruent?