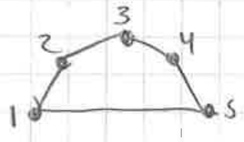
Last time : Pseudotriangulations

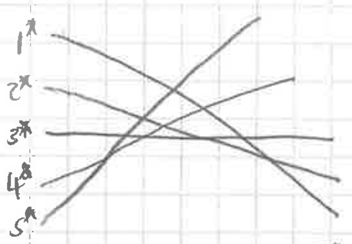
Today : Pseudoline arrangements and pointed pseudotriangulations duality

Recall duality :

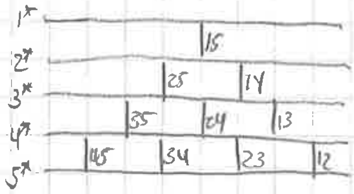
convex position



Point set P

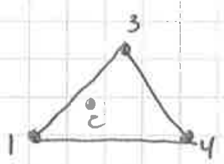


line arrangement P^*

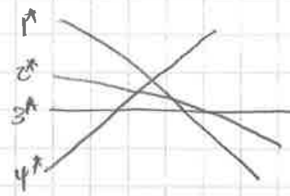


Sorting network $\mathcal{N}_P$

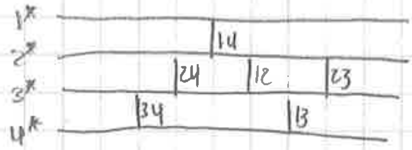
general position.



P



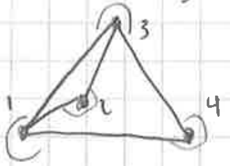
P^*



$\mathcal{N}_P$

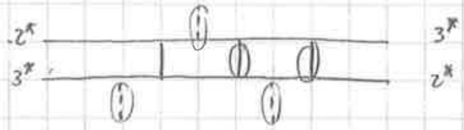
Today :

pointed vertex angle $> 180°$



pointed pseudotriangulation

$\longleftrightarrow$ bijection



pseudoline arrangements supported at $\mathcal{N}_P^{*1}$?
(removing first and last levels)

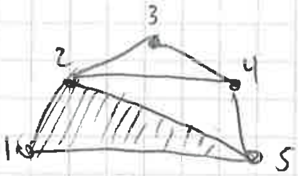
Theorem (Pilaud-Pocchiola 2012)

Let P be a set of points in general position (no three on a line, no two on a vertical line). Then, a set T of edges of P forms a pointed pseudotriangulation iff removing the commutators T^* (dual to T) from $\mathcal{N}_P$ yields a pseudoline arrangement supported at $\mathcal{N}_P^{*1}$

Remark : For points P in convex position, this recovers the duality between triangulations of P and pseudoline arrangements supported at $\mathcal{N}_P^{*1}$

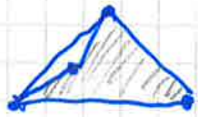
Proof sketch (convex position / general position)

Let P be a set of points in convex position. (general position.)



P

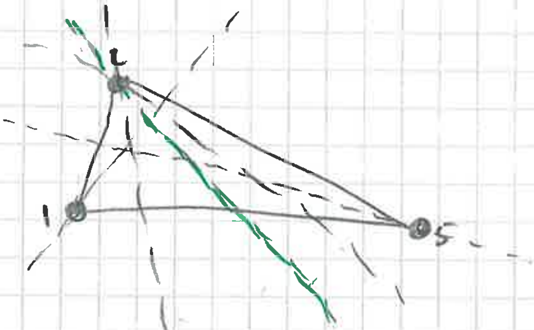
and a triangulation of P



P

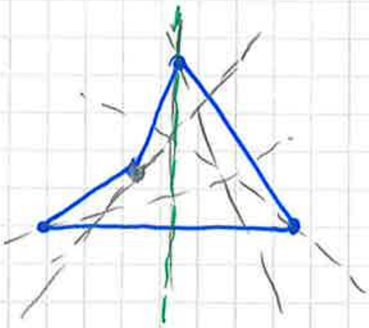
and a pseudotriangulation of P

A (strict) bisector of a triangle Δ is a line through one of its vertices that (strictly) separates the other two vertices



'bisectors'

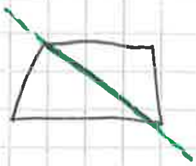
A pseudobisector of a pseudotriangle Δ

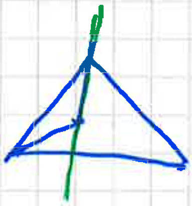


pseudobisectors

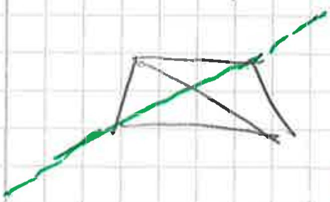
Observe:

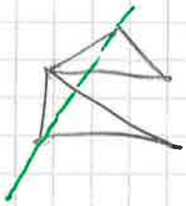
- The common edge of two adjacent triangles Δ_1 and Δ_2 (pseudotriangles) is a bisector (pseudobisector) of both

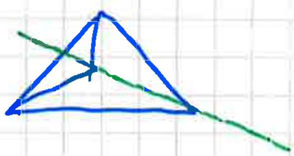




- Two triangles Δ_1 and Δ_2 of a triangulation (pseudotriangulation) have a unique common strict bisector







Further key observations Let T be a (pseudo) triangulation of P in convex position (generally)

(1) The dual set Δ^* of all (pseudo-) bisectors of a (pseudo-) triangle Δ of T is a pseudoline.

(2) The dual (pseudo-) lines Δ_1^* and Δ_2^* of any two (pseudo-) triangles Δ_1 and Δ_2 of T have a unique crossing point (the unique common strict (pseudo-)bisector of Δ_1 and Δ_2) and possibly a contact point (when Δ_1 and Δ_2 share a common edge)

(3) The set $T^* := \{\Delta^* : \Delta$ is a (pseudo-) triangle of $T\}$ is a pseudoline arrangement with contact points

(4) T^* covers P^* minus its first and last levels.

As a consequence, (pseudo-) triangulations yield pseudoline arrangements that cover P^* minus its first and last levels. (or N_P^{*1})

For the converse, it suffices to show that flips on (pseudo-) triangulations correspond to flips on pseudoline arrangements supported on N_P^{*1}