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Institute of Geometry

Seminarvortrag

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Seminarraum 2 Geometrie

Dowker's theorem for higher-order relations

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Given a binary relation between two sets, Dowker's Theorem (1952) asserts that the homology groups of two associated simplicial complexes, now known as Dowker complexes, are isomorphic. One modern form of the theorem, due to Brun and Salbu, interpolates a third space, a 'rectangle complex' which is functorially homotopy equivalent to both Dowker complexes.

What can be said about relations defined on three or more sets? For an order m relation, it turns out that there are functorial homotopy equivalences between a 'cuboid complex' and m higher-order Dowker complexes. There are many proof strategies and many different geometric models for the spaces in the theorem. We will follow a proof strategy that uses Smale's homotopy mapping theorem, via a 'cellular Dowker lemma' that expresses the main idea in general form.

If time permits, I will conclude the talk with a detailed discussion of order-3 relations, identifying seven functorially defined homotopy types and twelve natural transformations between them.

This is joint work with Chad Giusti, Vladimir Itskov, Michael Robinson, Radmila Sazdanovic, Nikolas Schonsheck, Melvin Vaupel, Iris Yoon, originating in a breakout session at a workshop hosted by the American Institute of Mathematics.

Michael Kerber