

Institut f. Geometrie

Geometry Seminar

Wednesday 26.6.2024, 13:45

Seminarraum 2, Kopernikusg. 24/IV

Homotopy and singular homology groups of finite (di)graphs

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We extend classical results in algebraic topology for higher homotopy groups and singular homology groups of pseudotopological spaces. Pseudotopological spaces are a generalization of topological spaces that also include graphs and directed graphs. More specifically, we show the existence of a long exact sequence for homotopy groups of pairs of closure spaces and that a weak homotopy equivalence induces isomorphisms for homology groups.

Our main result is the construction of a weak homotopy equivalence between the geometric realizations of (directed) clique complexes and their underlying (directed) graphs. This implies that singular homology groups of finite graphs can be efficiently calculated from finite combinatorial structures, despite their associated chain groups being infinite dimensional. This work is similar to the work McCord did for finite topological spaces, but in the context of pseudotopological spaces. Our results also give a novel approach for studying (higher) homotopy groups of discrete mathematical structures such as digital images.

Kristóf Huszár