Visualizing and Exploring Planar Algebraic Arrangements – a Web Application

http://exacus.mpi-inf.mpg.de

Pavel Emeliyanenko
Max-Planck-Institut für Informatik
66123 Saarbrücken, Germany
asm@mpi-inf.mpg.de

Michael Kerber
Max-Planck-Institut für Informatik
66123 Saarbrücken, Germany
mkerber@mpi-inf.mpg.de

Abstract

A web application is presented to compute, plot, and interactively explore planar arrangements induced by algebraic plane curves of arbitrary degree. It produces accurate curve plots and reflects the exact topology for any arrangement, including degenerated cases. Various user interface features allow the interactive exploration of the arrangement structure. This makes the tool useful for demonstrative and educational purposes, especially as it runs without initial installation process.

Categories and Subject Descriptors:
I.3.5 [**Computer Graphics**]: Computational Geometry and Object Modeling—*Geometric algorithms*; G.4 [**Mathematical Software**]—*User interfaces*

General Terms: Algorithms, Experimentation

Keywords: Arrangements, algebraic curves, plotting, exact geometric computation

1. INTRODUCTION

It is generally acknowledged that demonstrative advertisement programs are essential for the success of a project, be it business- or research-oriented. The xAlci web application, which is presented in this video, is part of Exacus[1], a collection of libraries containing exact geometric algorithms for curves and surfaces. Our main contribution is to bring recent research in this area to a broader community.

In particular, the web program combines two algorithms that were recently implemented within the Exacus project: algorithm to compute the exact arrangement induced by real algebraic plane curves in the plane [2] [3], and algorithm to accurately draw arcs of algebraic plane curves [4]. Altogether, the algorithms produce topologically correct and geometrically accurate plots of algebraic arrangements. Besides the mere plot, combinatorial information about the arrangement is provided – this allows to identify interesting regions which can be interactively explored by zooming into them. It is worth mentioning that all degenerate cases (e.g., isolated vertices, vertical asymptotes, tangential intersections) are handled exactly.

The computations are performed on a web server. The combinatorial arrangement description, and the image (in png format) are sent to the client who displays the result in the web browser using Flash[2]. With this architecture, the user is not burdened with installing additional software to use our application.

[1]http://www.mpi-sb.mpg.de/projects/EXACUS

SCG'08, June 9–11, 2008, College Park, Maryland, USA.
ACM 978-1-60558-071-5/08/04.

2. USING THE PROGRAM

xAlci's web application has undergone a very thorough change since the first version released. We tried to consider all remarks and suggestions from the Exacus team to make the interface intuitively clear and easily extensible. We summarize the features of xAlci web application as follows:

- polynomials are given in MAPLE format with rational coefficients
- zoom, pan and focus on arbitrary portion of the curve plot
- highlighting arrangement features (arcs, vertices and faces)
- display additional data such as the number of arrangement faces, edges, vertices and isolated vertices
- choose one of three visualization modes: visualize complete arrangement, visualize in flat color or visualize selected arcs only
- export arrangement plots as a png image
- Macromedia Flash Player 8.0 compliance

We consider the program usage step-by-step. In order to compute an arrangement, the user enters a list of polynomials, delimited by comma to the input area and clicks the **Analyse curves** button. xAlci computes the arrangement induced by the vanishing loci of these polynomials and fills the lists of arcs, vertices, and faces. Each arc is given by the x-coordinates and arc numbers[3] of its end-points, and the arc number in its interior (the algorithm produces arcs such that the internal arc number is unique for all arcs). $\pm\infty$ instead of an arc number for an endpoint indicates that the arc has a vertical asymptote at the corresponding x-coordinate.

By clicking the **Plot curve arcs** button, the arrangement is plotted in the main frame. Also, the visualization tools listed below become available:

 : zoom in by a factor of 2

[2]http://www.macromedia.com/software/flash/about/
[3]The *arc number* of a point with x-coordinate x_0 on curve f defines its index in the order amongst all points of f with $x = x_0$

 : zoom out by a factor of 2

 : reset to default zoom

 : focus on point – click the curve plot to center at a special point

 : focus on region – hold mouse button and select the area of interest

 : feature selection mode

The last item deserves special attention. In large arrangements consisting of thousands of arcs, it is hard to find a certain arrangement item on the curve plot – this is where *feature selection mode* might be useful. Clicking the curve plot in this mode executes a *point location query* [6] and a corresponding item in the face list gets immediately highlighted. Alternatively, one can highlight any arrangement items(s) by selecting them in the arcs, vertices or faces lists.

Arrangements can be rasterized using one of the following visualization modes:

complete : Plots the whole arrangement. Each curve is drawn in its own color; for a single curve, each arcs has a different color. Critical points and curve intersections are marked by black dots.

complete one-color : As before, but end-points are not marked and single curves are draw in a flat color.

selected arcs : Plots only arcs that are selected in the arc list. Colors are assigned as in the first mode.

3. ALGORITHMIC BACKGROUND

What happens in detail when the user hits the **Analyse curves** button? Two steps are performed:

- Segmentation of each input curve into a list of x-monotone arcs [3]. This is done by computing the topology of the algebraic curve together with geometric information about the critical points of the curve, using a symbolic-numeric method, The output of this *curve analysis* resembles a *cylindrical algebraic decomposition* [1], from which the x-monotone arcs are read off easily.
- Arrangement computation of the curve arcs [2]. In that step, we apply the generic sweep-line implementation from CGAL's[4] `Arrangement_2` package [6] on the set of x-monotone arcs computed in the first step. We instantiate the package so that algebraic curves of any degree are handled. For that, we provide a suitable *Traits-class* based on efficient methods to analyze a single curve (see previous item), and a pair of algebraic curves.

When pressing the **Plot curve arcs** button each curve arc from an arrangement is rasterized separately [4] as follows:

- A curve arc is traced in both directions starting from a seed point which is guaranteed to lie on this arc. Each time the algorithm has 8 directions (pixels) to follow. Using range analysis [5] and some heuristics one can quickly decide which direction to take. In order to distinguish closely located curve branches we employ a local space subdivision. The curve branches are not discriminated beyond certain threshold (which is within the current pixel size) when the direction of tracing can be determined uniquely. This results in a substantial speed-up for curves having a lot of closely located features.

[4]http://www.cgal.org

All three sub-algorithms guarantee the correctness of their results, such that the resulting plot is a correct approximation of an arrangement with a given resolution. This is true for any input, as the algorithms are designed to handle all special cases. Moreover, efficiency is attained by avoiding expensive algebraic computations in the algorithms, and applying approximate methods instead to speed up the computation, without sacrificing the exactness of the overall result. Please refer to [3], [2], [4] for more explanations.

4. ABOUT THE VIDEO

In the video we mainly focus on the usage of the web application, demonstrate most of its capabilities, and give a short overview about the techniques used for the visualization of algebraic curves.

Those parts of the video that present the webdemo were produced by *screen capturing* using *Virtual dub*[5] free software. The final make-up and overlaying the video with audio tracks was done using *Adobe Premiere Pro*[6].

Acknowledgements

We are indebted to the whole EXACUS team, in particular to Eric Berberich and Michael Sagraloff, for thoroughly testing the application and useful suggestions to extend and improve the web interface.

5. REFERENCES

[1] B. F. Caviness, J. R. Johnson (eds.): *Quantifier Elimination and Cylindrical Algebraic Decomposition, Texts and Monographs in Symbolic Computation.* Springer, 1998.

[2] A. Eigenwillig, M. Kerber: "Exact and Efficient 2D-Arrangements of Arbitrary Algebraic Curves". In: *Proc. of the Nineteenth Annual ACM-SIAM Symp. on Discrete Algorithms (SODA08)*, 2008 122–131.

[3] A. Eigenwillig, M. Kerber, N. Wolpert: "Fast and Exact Geometric Analysis of Real Algebraic Plane Curves". In: C. W. Brown (ed.) *Proocedings of the 2007 International Symp. on Symbolic and Algebraic Computation (ISSAC 2007)*, 2007 151–158.

[4] P. Emeliyanenko: *Visualization of Points and Segments of Real Algebraic Plane Curves.* Master's thesis, Saarland University, 2007.

[5] F. Messine: "Extension of Affine Arithmetic: Application to Unconstrained Global Optimization". *Journal of Universal Computer Science* **8** (2002) 992–1015.

[6] R. Wein, E. Fogel, B. Zukerman, D. Halperin: "2D Arrangements". In: *CGAL-3.3 User and Reference Manual*, 2007. URL `http://www.cgal.org/Manual/3.3/doc_html/cgal_manual/Arrangement_2/Chapter_main.html`.

[5]http://www.virtualdub.org/

[6]http://www.adobe.com/de/products/premiere/