ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Dynamic 2D/3D Registration

Sofien Bouaziz   Andrea Tagliasacchi

http://lgg.epfl.ch/publications/2014/2d3dRegistration

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Presenters

Dr. Sofien Bouaziz

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE | faceshift

Dr. Andrea Tagliasacchi

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE | University of Victoria

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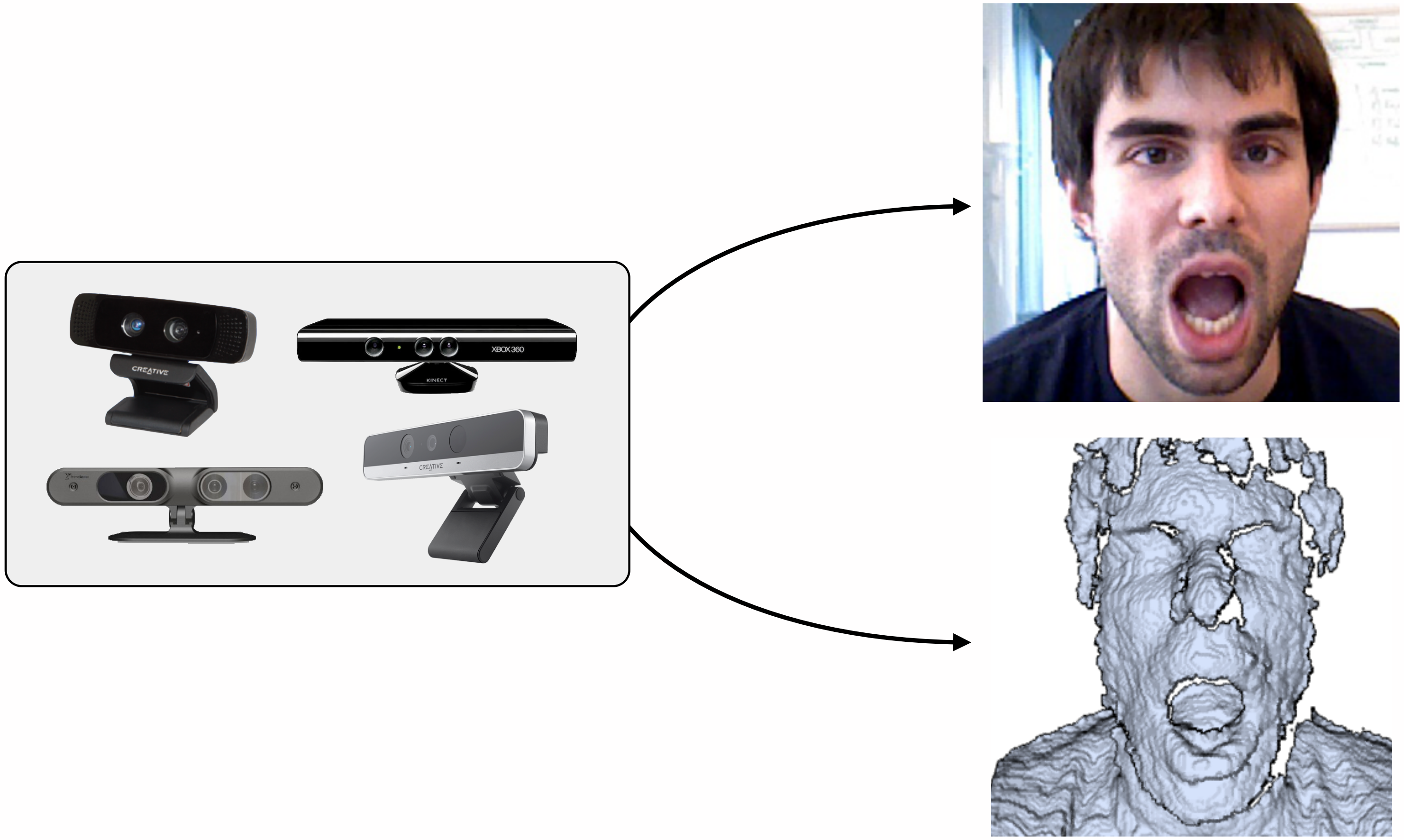
RGB-D Sensors



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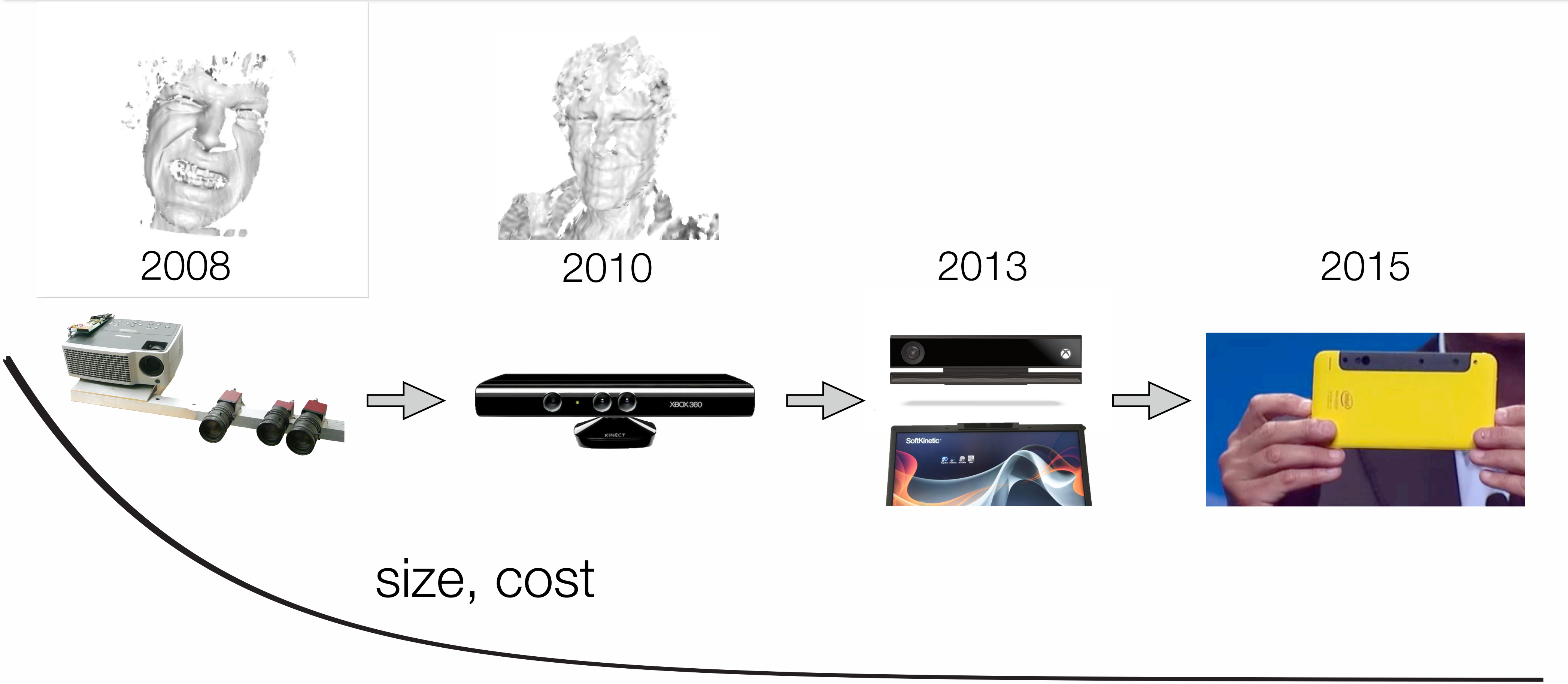
RGB-D Sensors



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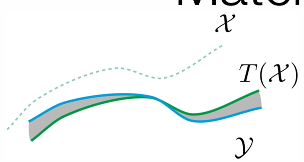
Outline

Theory

$$\arg\min_T \; E_{\text{match}} + \sum_i w_i E^i_{\text{prior}}$$

Matching



$$\int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

Regularizing

Demos and code

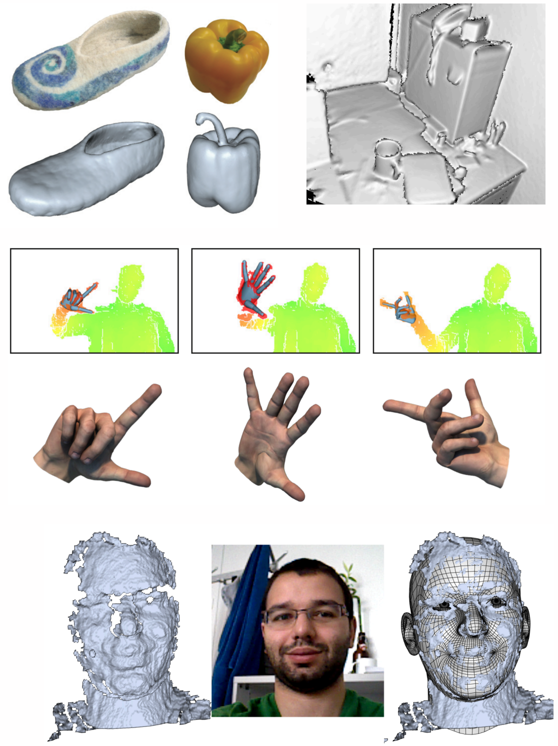
faceshift

SKANECT

MATLAB

GitHub
OpenGP/htrack

Applications



Overview


Introduction (5min)


- Registration (5 min)


  - 3D Geometry (25 min)


  - 2D Images (10 min)


  - Combined 2D/3D (5 min)


  - Robust Registration (15 min)


  - Q&A (5min)


- Applications


  - Rigid Scanning (10 min)


  - Articulated Tracking (10 min)


  - Non-rigid Modeling (10 min)


  - Realtime Face Tracking (10 min)


  - Q&A (5min)


Outlook (5 min)

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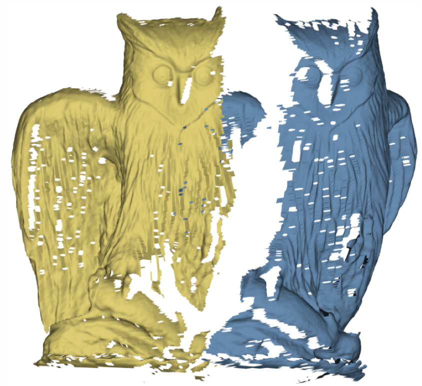
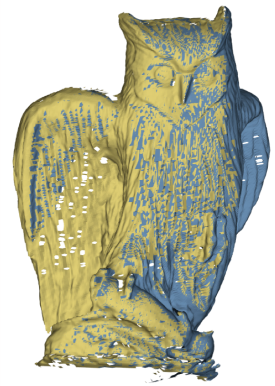
Outlook (5 min)

Registration - Examples

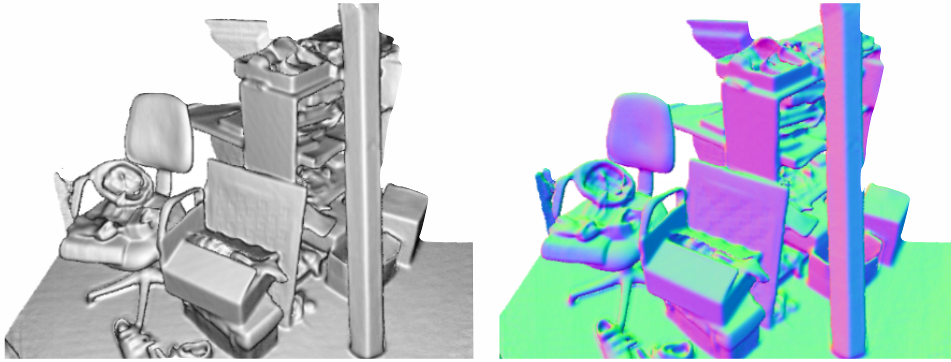
- Scan to scan

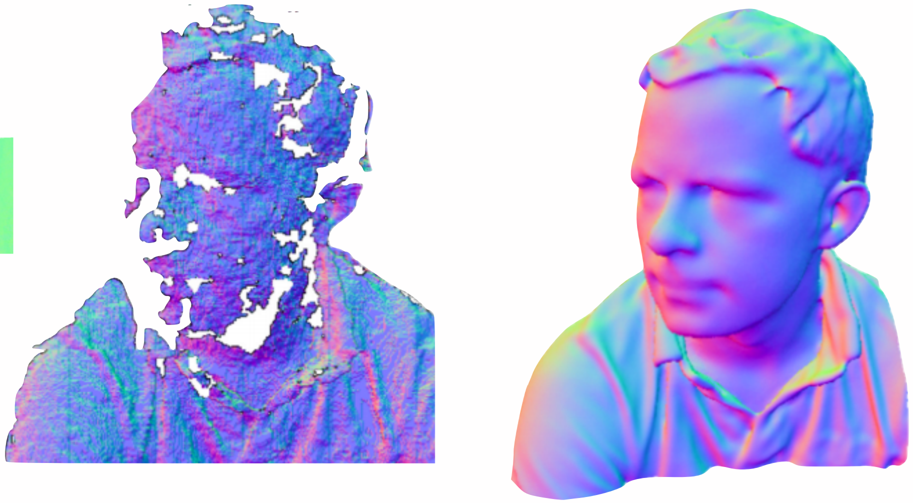




Registration - Examples

- Scan to scan

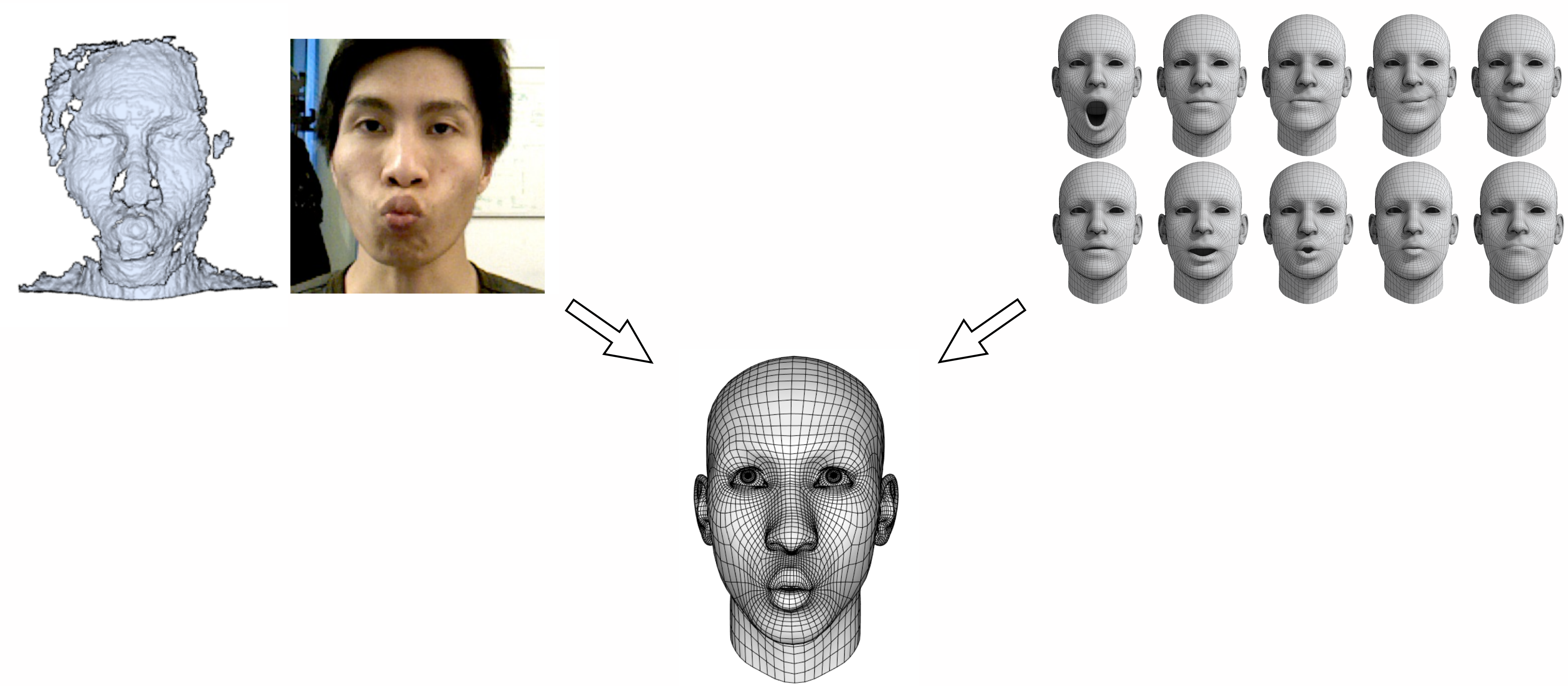




Newcombe, Izadi, Hilliges, Molyneaux, Kim, Davison, Kohli, Shotton, Hodges, Fitzgibbon:
KinectFusion: Real-Time Dense Surface Mapping and Tracking, *ISMAR 2011*

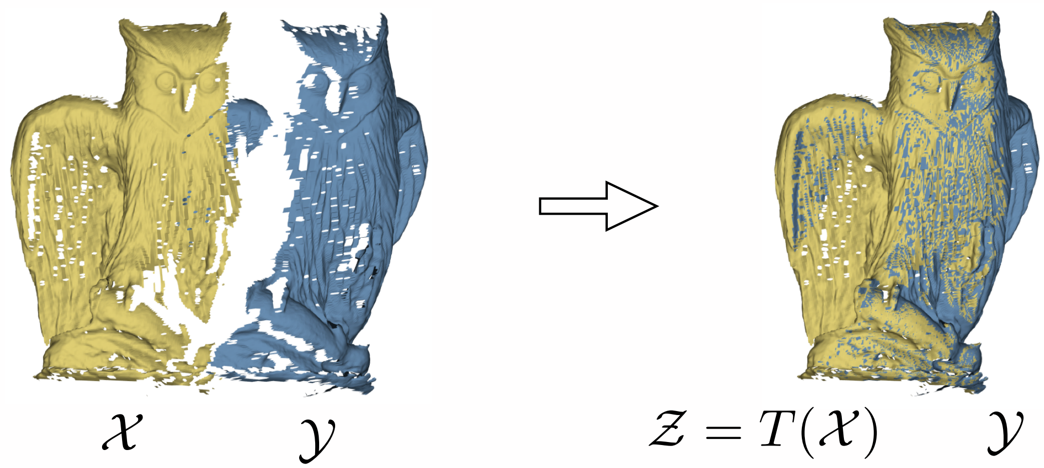
Registration - Examples

- Parameterized template to scan



Pairwise Registration

- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
  - find a *transformation* $T(\mathcal{X})$ that brings $\mathcal{X}$ into alignment with $\mathcal{Y}$



Pairwise Registration

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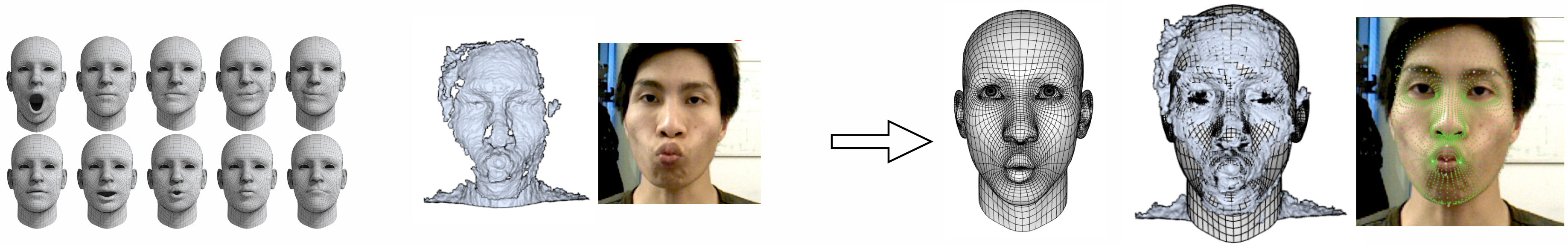

$\mathcal{X}$ $\mathcal{Y}$ $\mathcal{Z} = T(\mathcal{X})$

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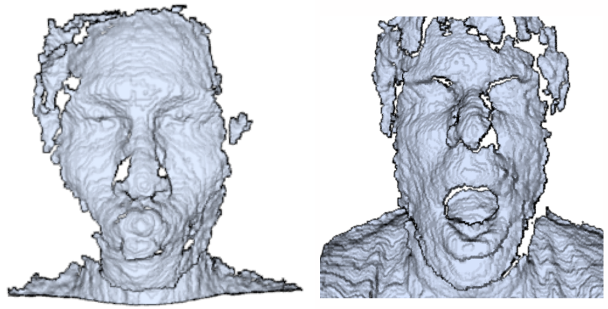
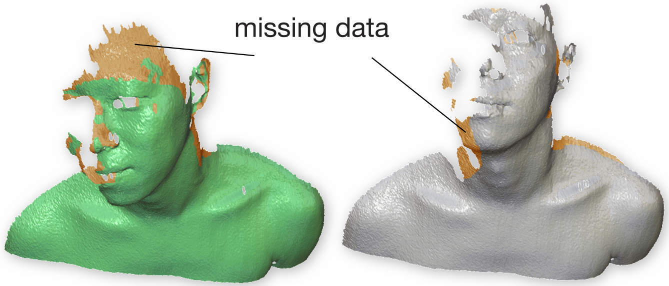
Pairwise Registration

- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
  - find a *transformation* $T(\mathcal{X})$ that brings $\mathcal{X}$ into alignment with $\mathcal{Y}$
- Two main questions:
  - How do we measure the quality of the alignment?
  - What transformations are acceptable?

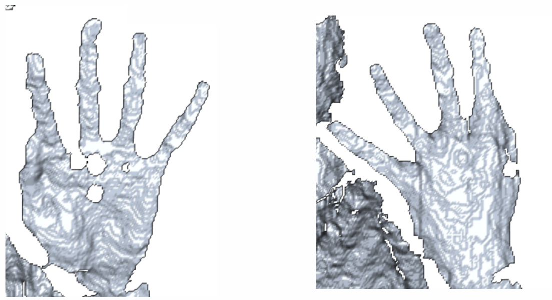
Pairwise Registration

- Issues


Noise



Partial matching


Ambiguity


Illumination changes

Registration

- Registration as energy minimization

source

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

transformation

target

Registration

- Registration as energy minimization

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Registration

- Registration as energy minimization

$$\arg\min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Transformation Error

What transformations are allowed / good?

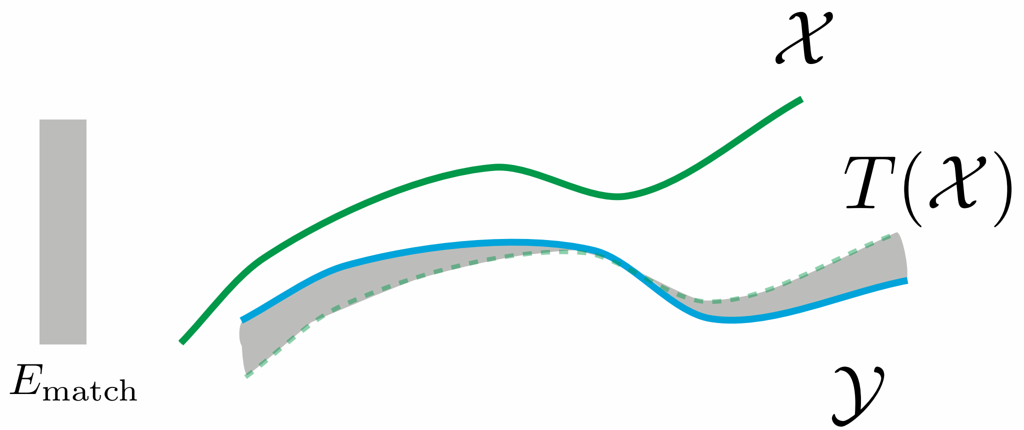
Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid

composite

articulated

elastic

fluid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion prior

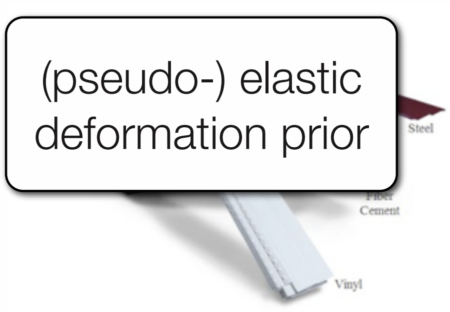
rigid

skeletal model prior

articulated

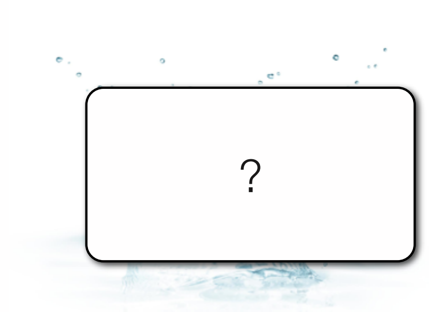
data-driven prior

composite



(pseudo-) elastic deformation prior

elastic



?

fluid

Registration

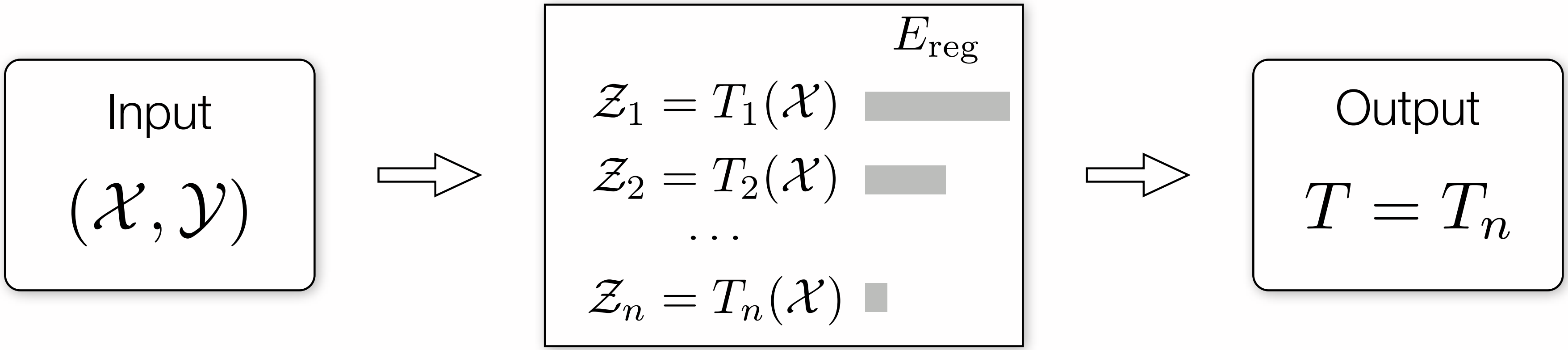
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization $\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$
  - typically non-linear
  - common solution: linearize, iterate

source

transformation

target



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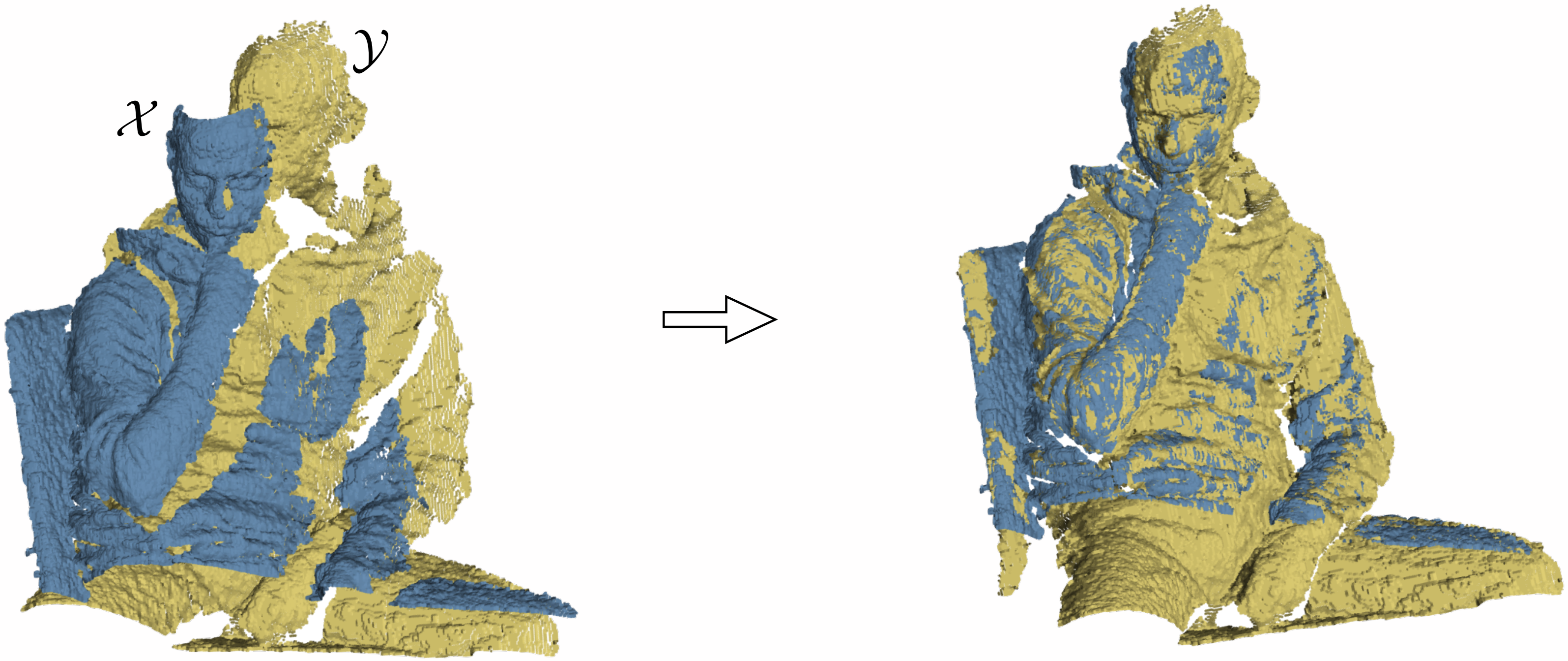
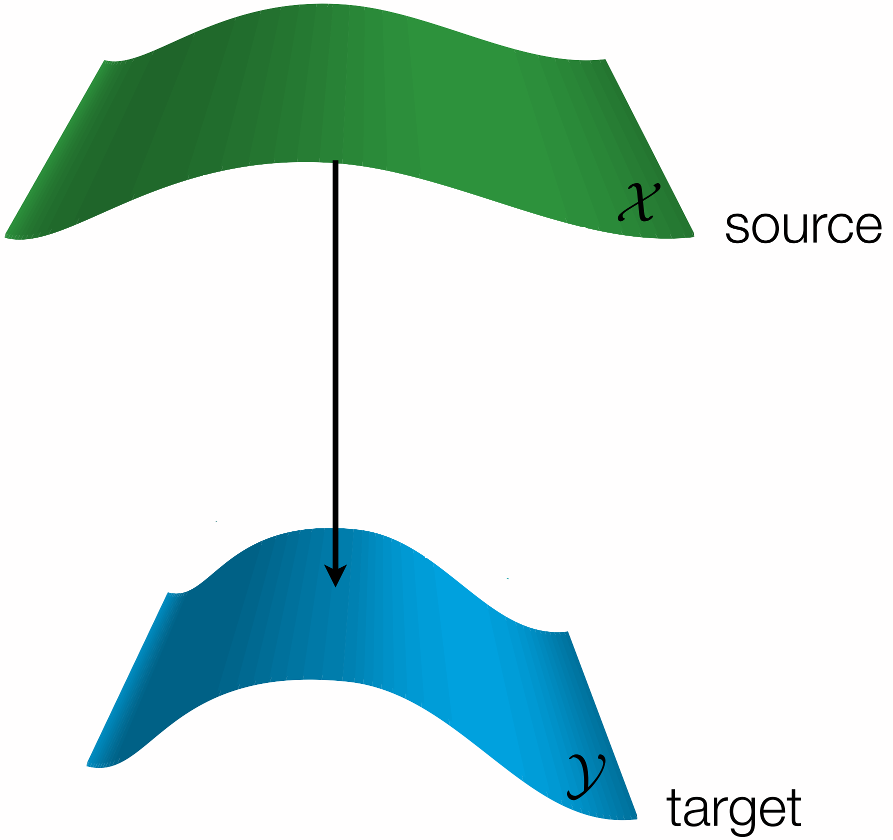
Outlook (5 min)

3D Registration



3D Registration



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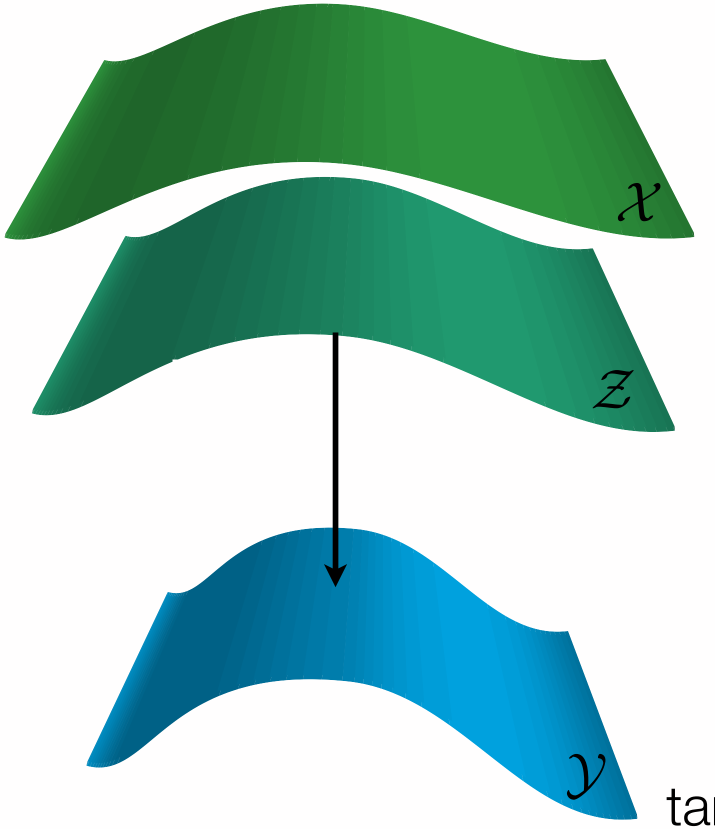
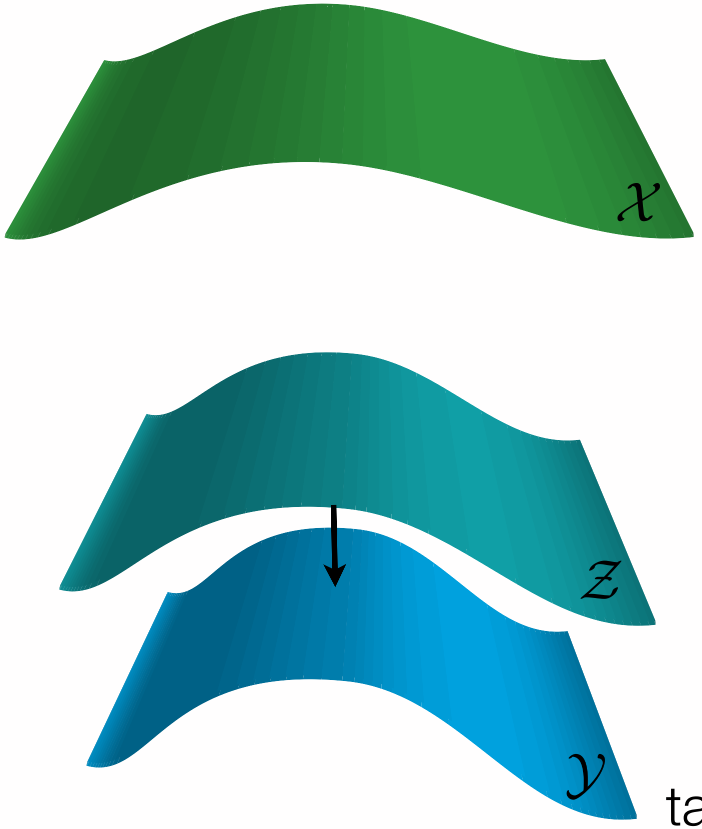
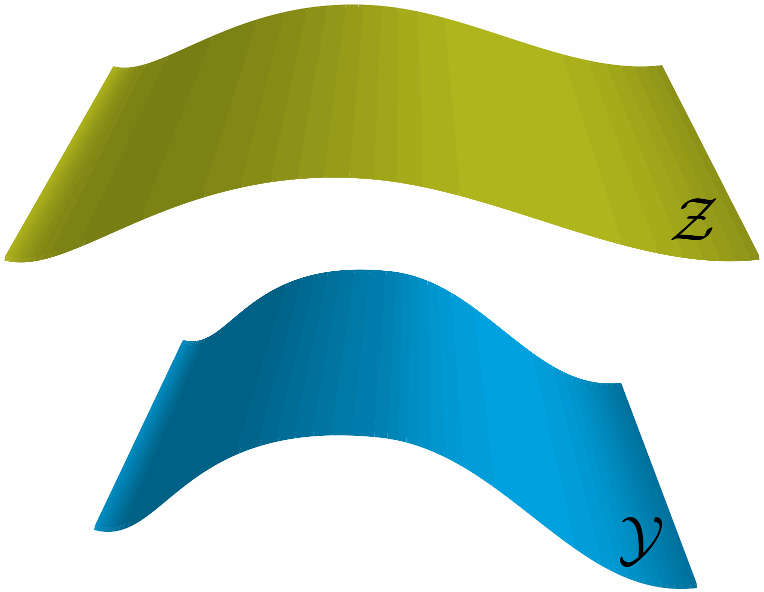
3D Registration



3D Registration



3D Registration - Matching

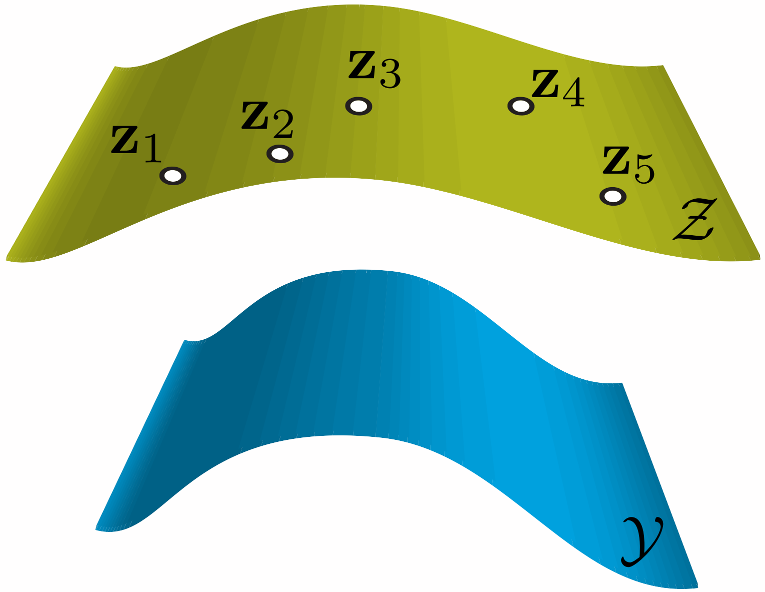


$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$

distance function

3D Registration - Matching



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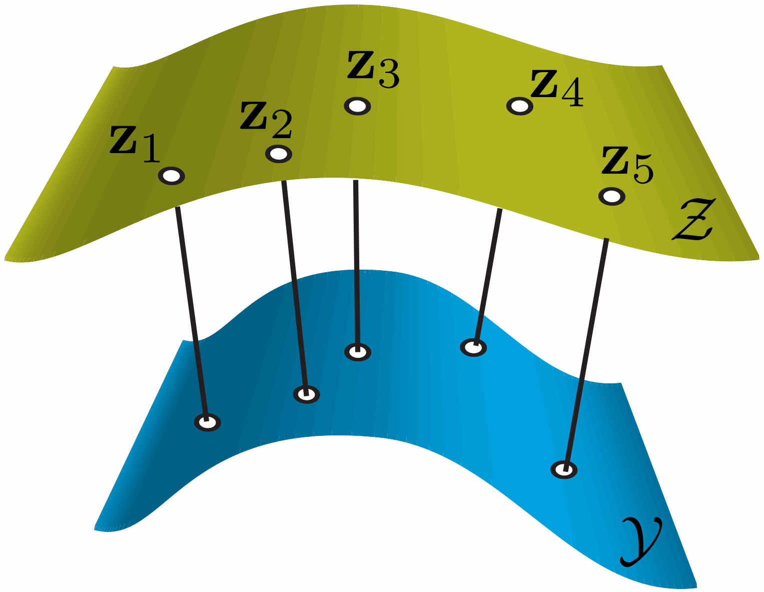

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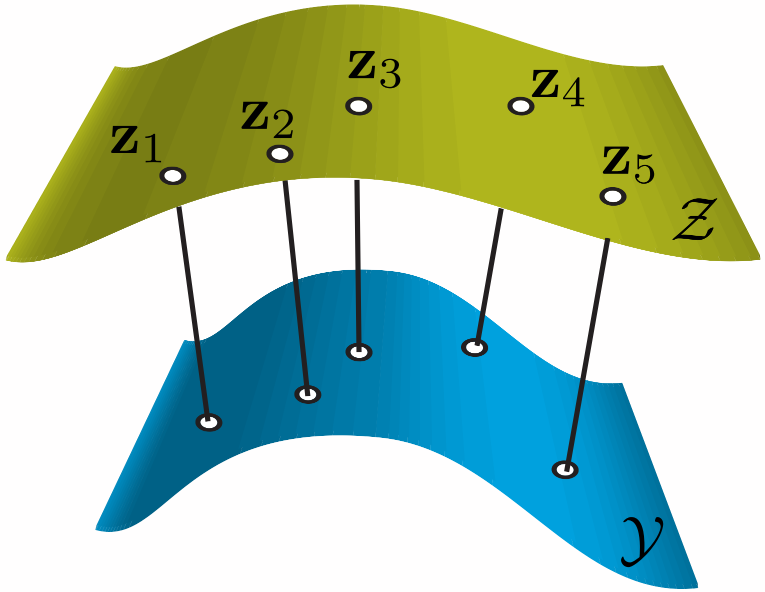


discretized matching cost based on point correspondences

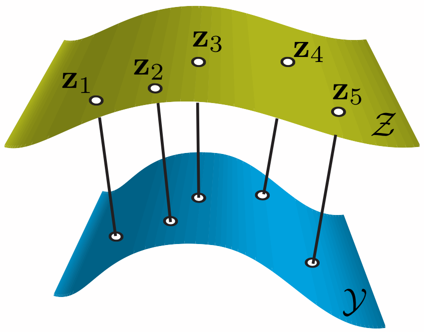
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$

⇓

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

3D Registration - Matching



correspondence weight

corresponding point on target

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

transformed point on source

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
  - **Step 1:** find correspondences using closest points for fixed transformation

    → efficient data structures
  - **Step 2:** find best rigid transformation for fixed correspondences

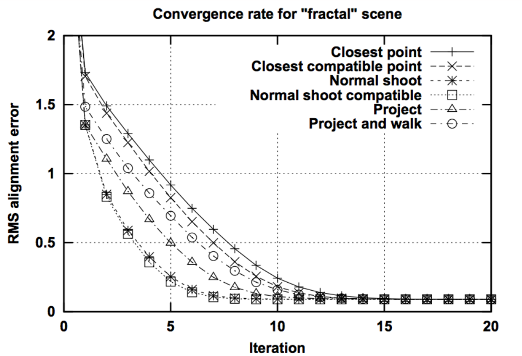
    → closed form solution

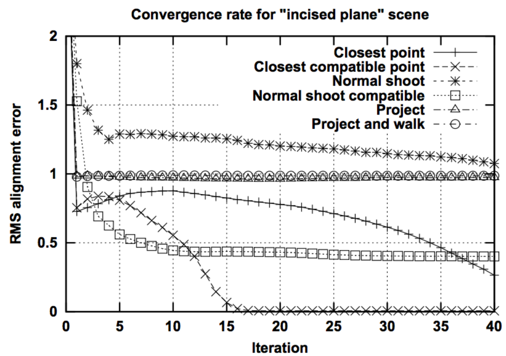
Iterate until convergence

DEMO

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
  - (approximate) closest points→ (more) **efficient** data structures
  - **weight** accounts for importance and confidence
  - **heuristics** to prune or down-weigh bad correspondences





Rusinkiewicz, Levoy: **Efficient Variants of the ICP Algorithm**, *3D Digital Imaging and Modeling, 2001*

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

- Side Remark: **Error norm**
  - squared Euclidean distance is sensitive to outliers
  - robust norms reduce this sensitivity

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid

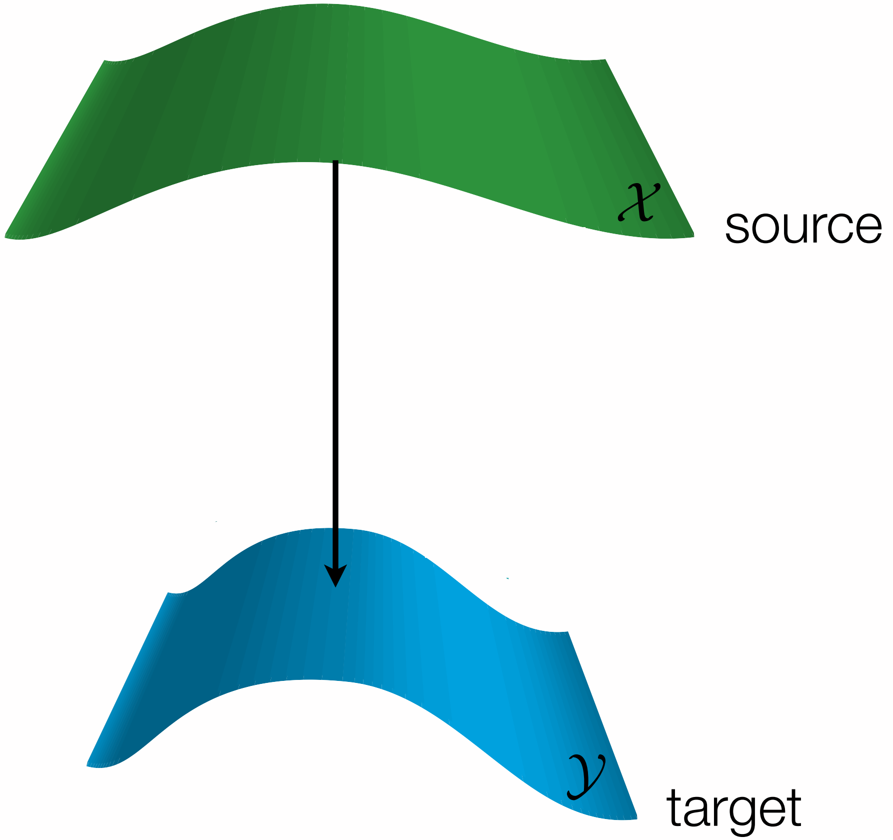
composite

articulated

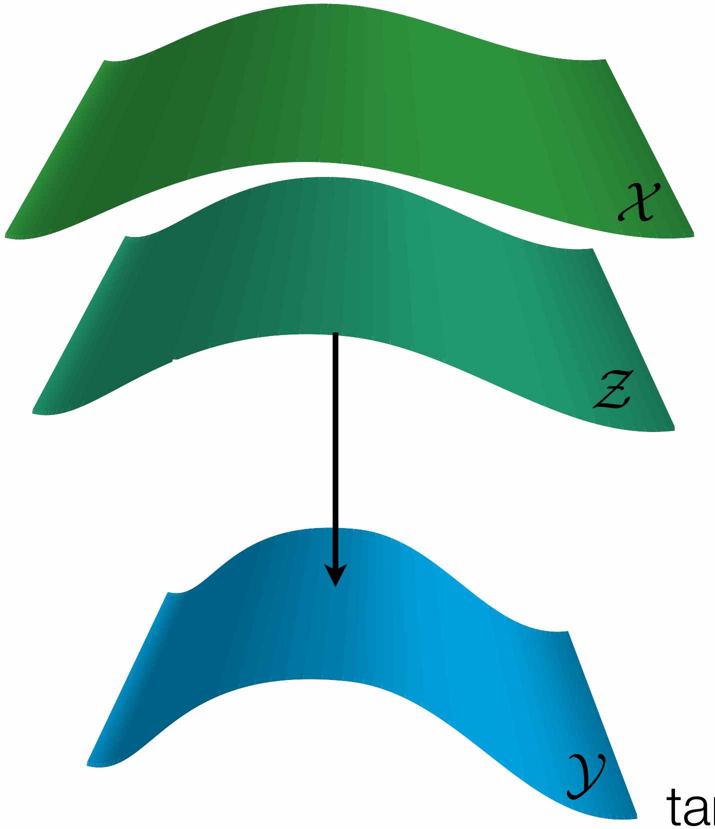
elastic

fluid

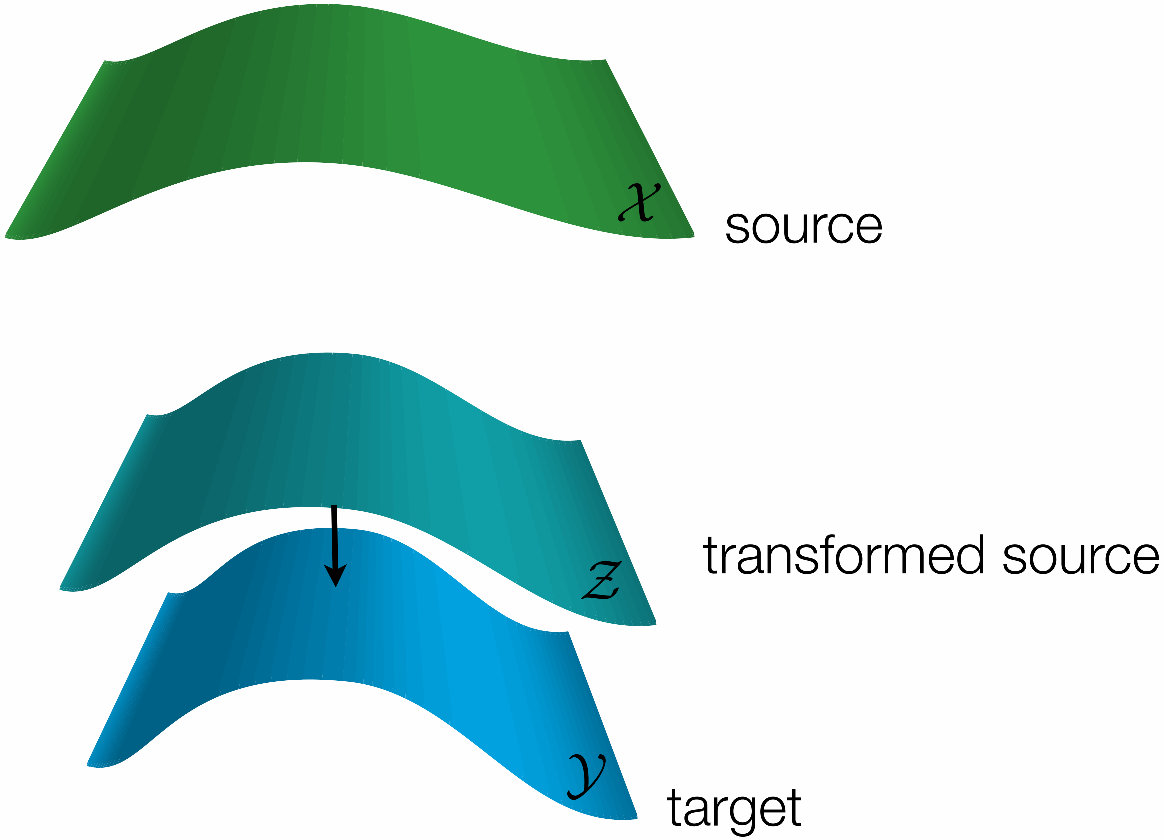
3D Registration - Prior



3D Registration - Prior



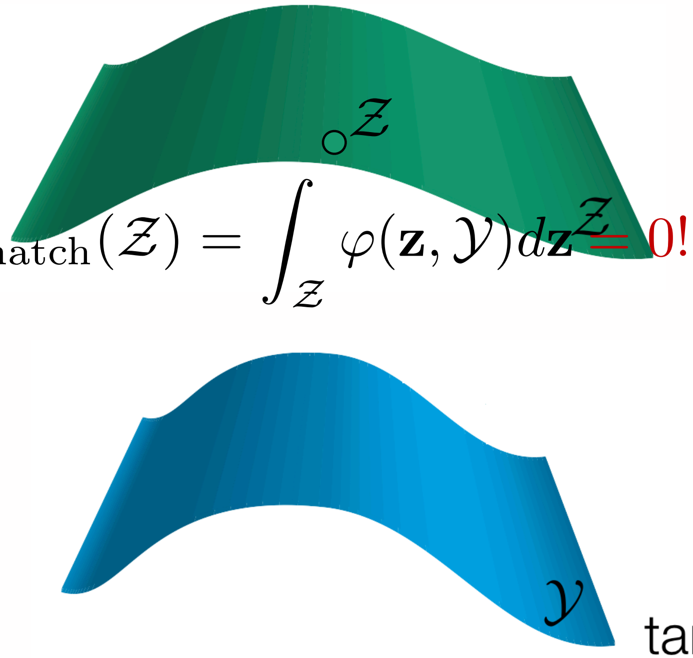
3D Registration - Prior



3D Registration - Prior

- Why priors?

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z} = 0!!!$$

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

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composite

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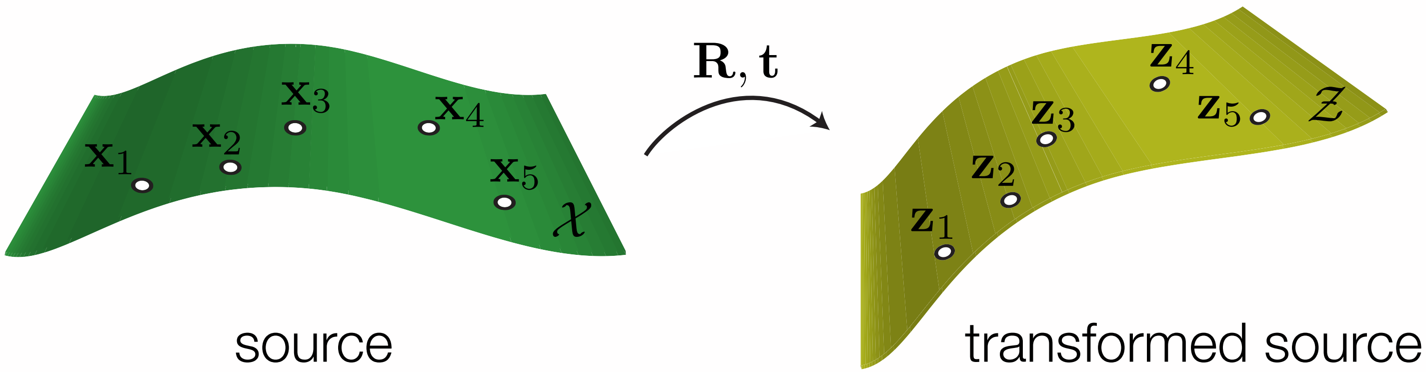
elastic

fluid

3D Registration - Prior

- Global Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



$$E_{\text{prior}}(Z, \mathbf{R}, \mathbf{t}) = \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

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composite

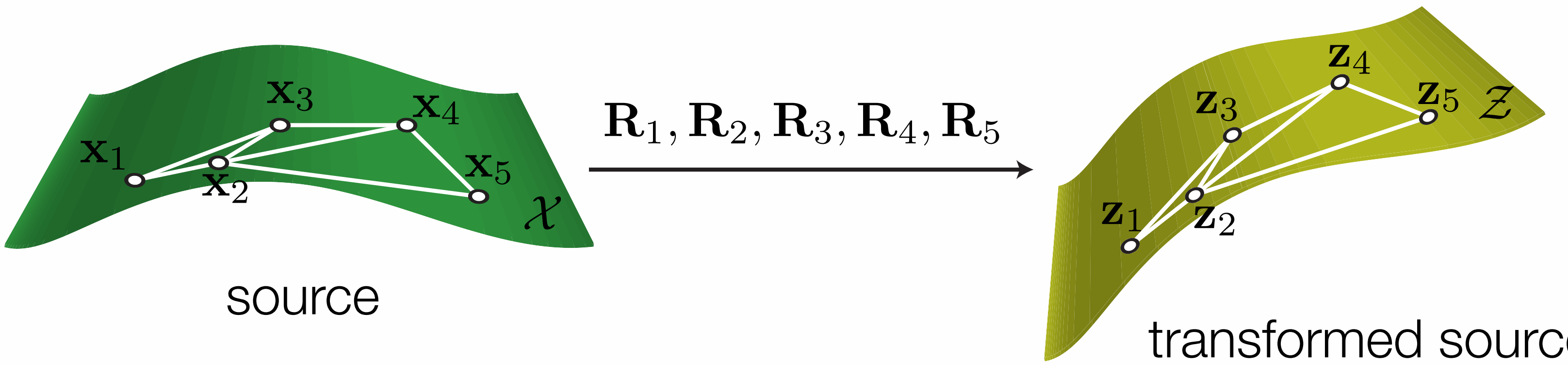
articulated

elastic

fluid

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

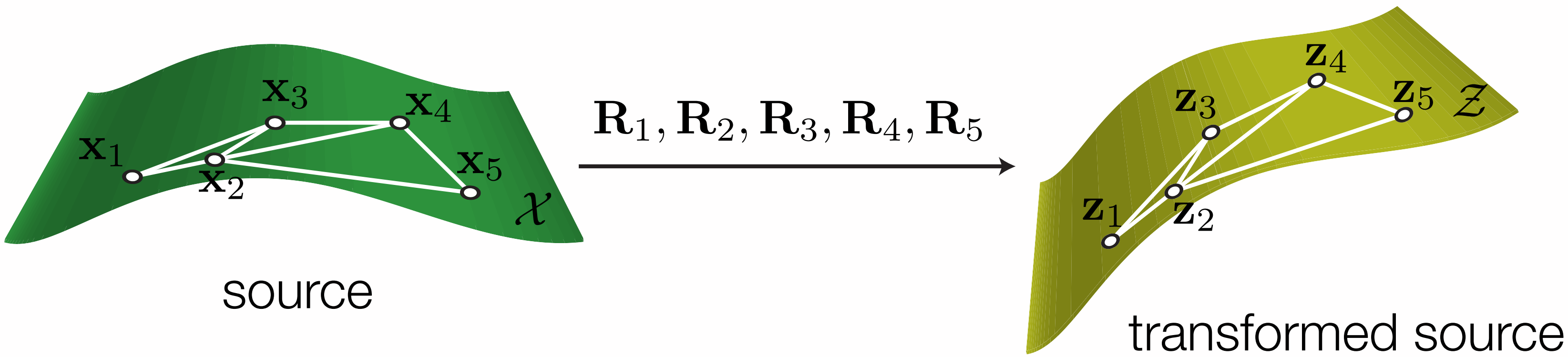


$$E_{\text{prior}}(Z, \{\mathbf{R}_i\}) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

Sorkine, Alexa: **As-Rigid-As-Possible Surface Modeling**, *SGP 2007*

3D Registration - Prior

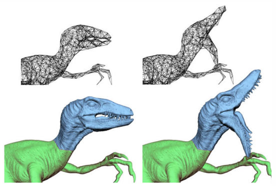
- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



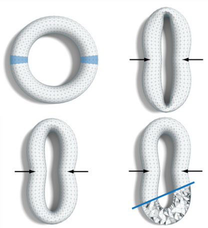
DEMO

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



Sumner, Schmid, Pauly:
Embedded Deformation for Shape Manipulation
ACM SIGGRAPH 2007



Bouaziz, Deuss, Schwartzburg, Weise, Pauly:
Shape-Up: Shaping Discrete Geometry With Projection
SGP 2012

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid

elastic

articulated

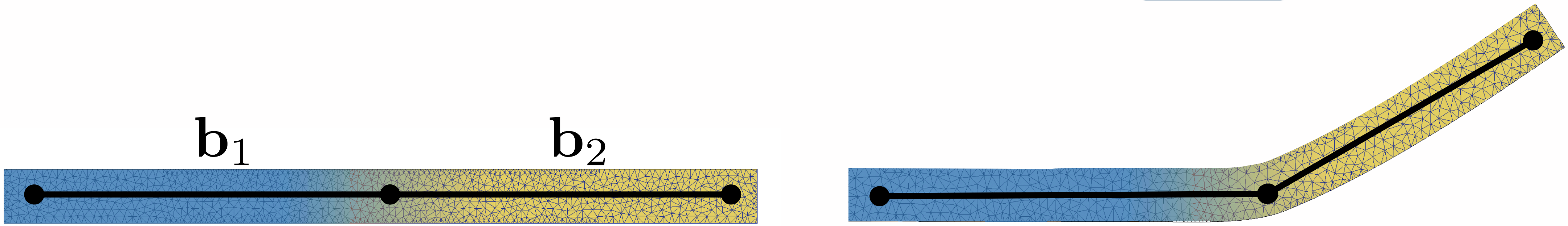
composite

fluid

3D Registration - Prior

- Linear Blend Skinning

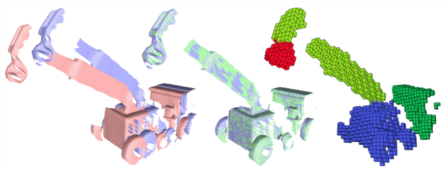
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$E_{\text{prior}}(Z, \mathbf{R}_j, \mathbf{t}_j) = \sum_{i=1}^{n} \sum_{j \in \mathcal{B}} w_{i,j} \|\mathbf{z}_i - (\mathbf{R}_j \mathbf{x}_i + \mathbf{t}_j)\|_2^2$$

blended

rigid transformations



Chang, Zwicker: **Range Scan Registration Using Reduced Deformable Models**
EUROGRAPHICS 2009

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid

composite

articulated

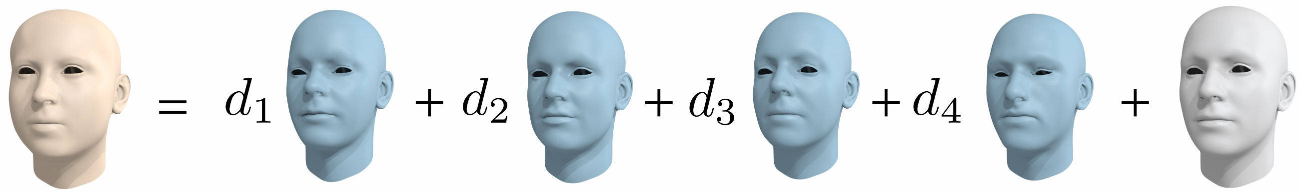
elastic

fluid

3D Registration - Prior

- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



coefficients

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

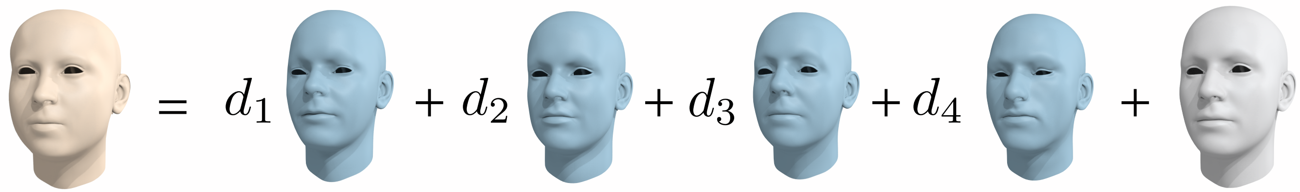
basis

mean shape

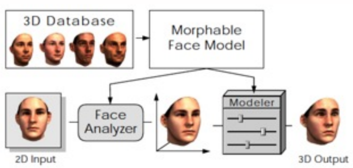
3D Registration - Prior

- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

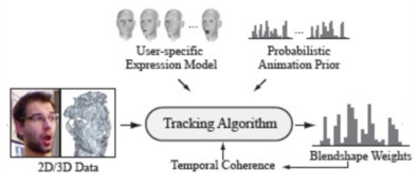


$$E_{\text{prior}}(Z, \mathbf{d}) = \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{P}(i,:)\mathbf{d} + \mathbf{m}(i))\|_2^2$$



Blanz, Vetter: **A Morphable Model for the Synthesis of 3D Faces**
ACM SIGGRAPH 1999

Weise, Bouaziz, Li, Pauly: **Realtime Performance-based Facial Animation**
ACM SIGGRAPH 2011



3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid

composite

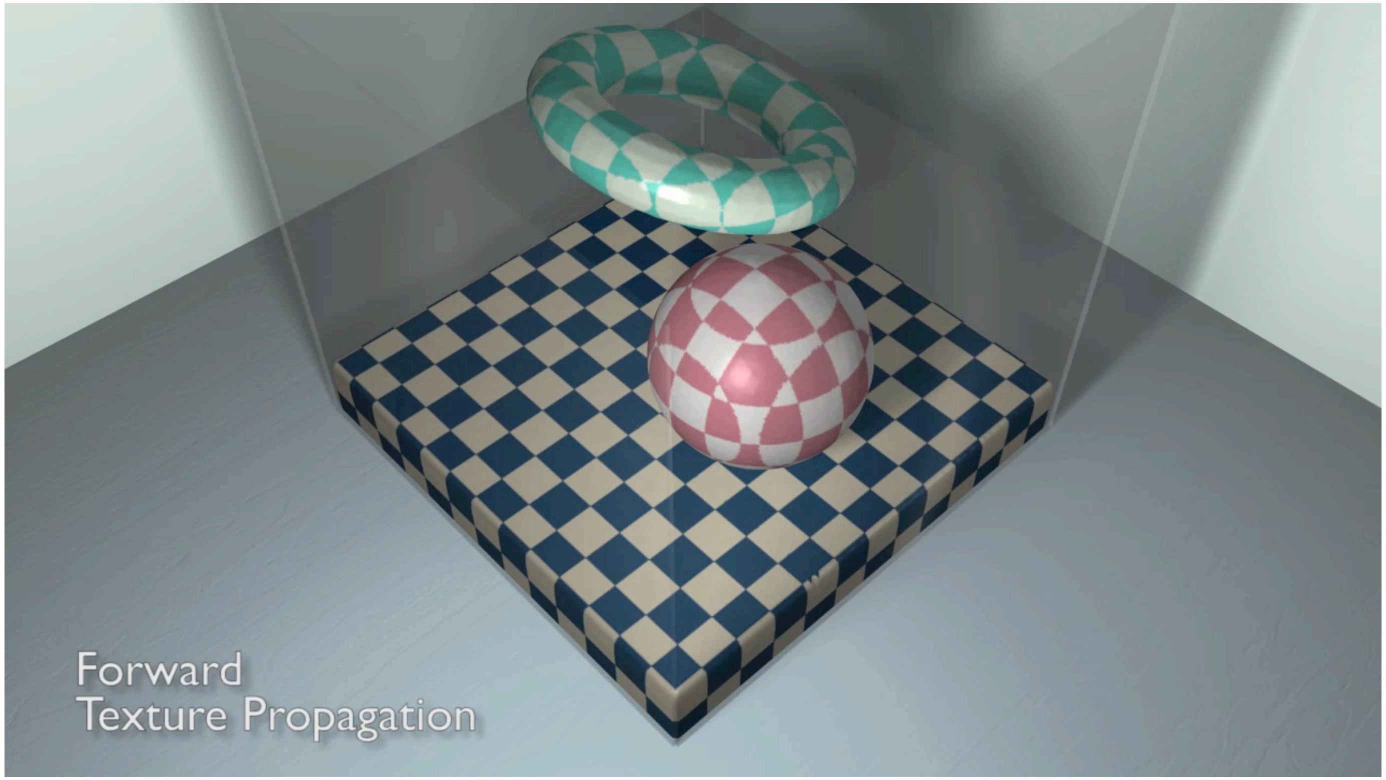
articulated

elastic

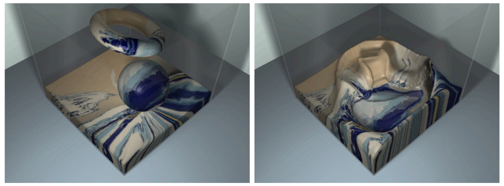
fluid

3D Registration - Prior

- Evolving topology

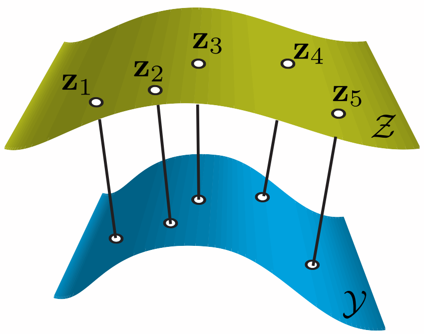


Bojsen-Hansen, Li, Wojtan:
Tracking Surfaces with Evolving Topolgy
ACM SIGGRAPH 2012



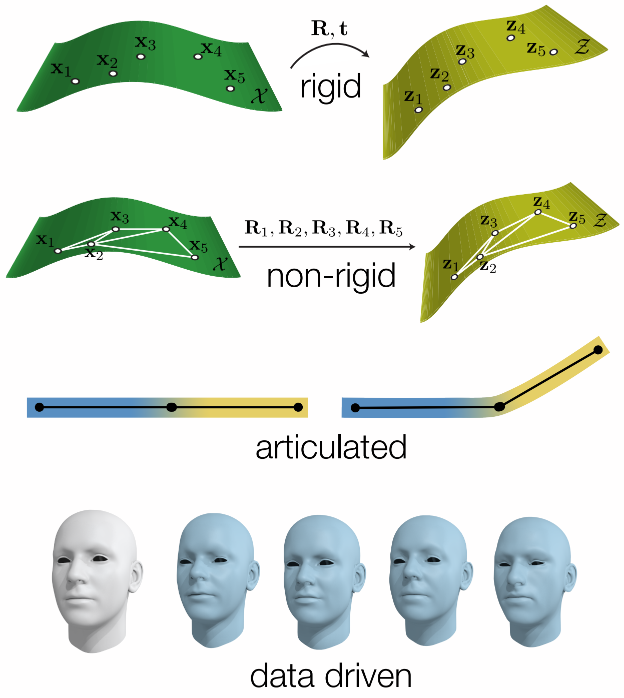
3D Registration - Recap

Matching



$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

Prior



Overview

Introduction (5min)

- Registration (5 min)
  - 3D Geometry (25 min)
  -  2D Images (10 min)
  - Combined 2D/3D (5 min)
  - Robust Registration (15 min)
  - Q&A (5min)
- Applications
  - Rigid Scanning (10 min)
  - Articulated Tracking (10 min)
  - Non-rigid Modeling (10 min)
  - Realtime Face Tracking (10 min)
  - Q&A (5min)

Outlook (5 min)

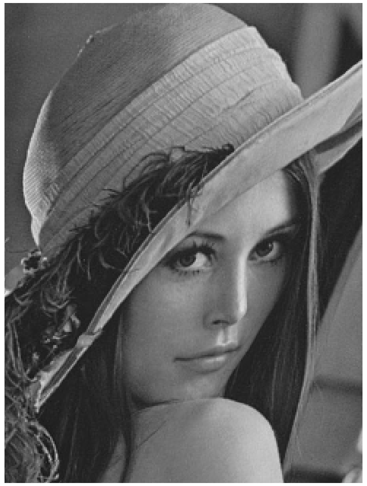
2D Registration

Source

$\mathbf{I}$

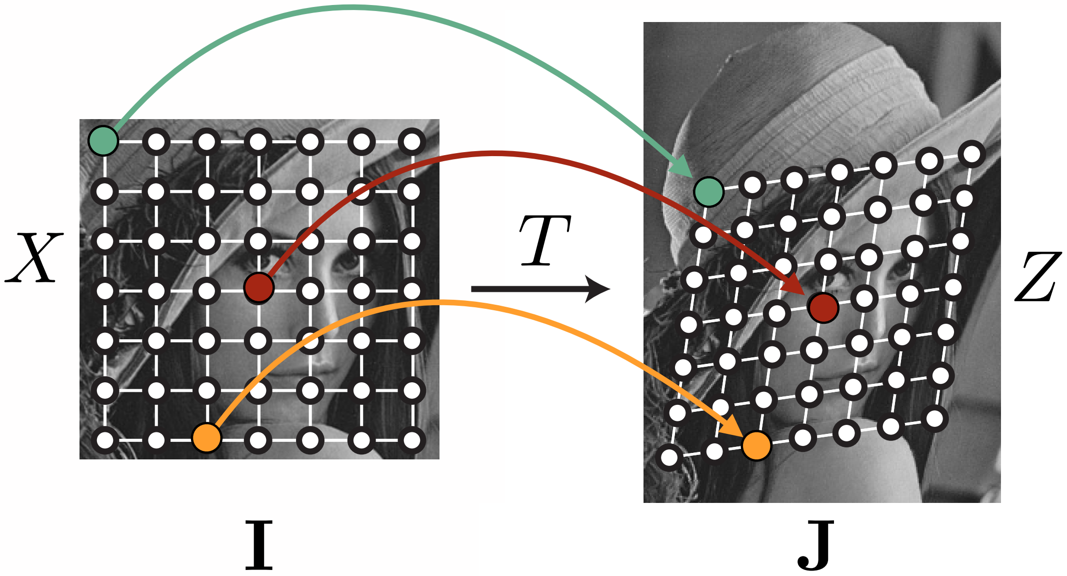
?

Target



$\mathbf{J}$

2D Registration

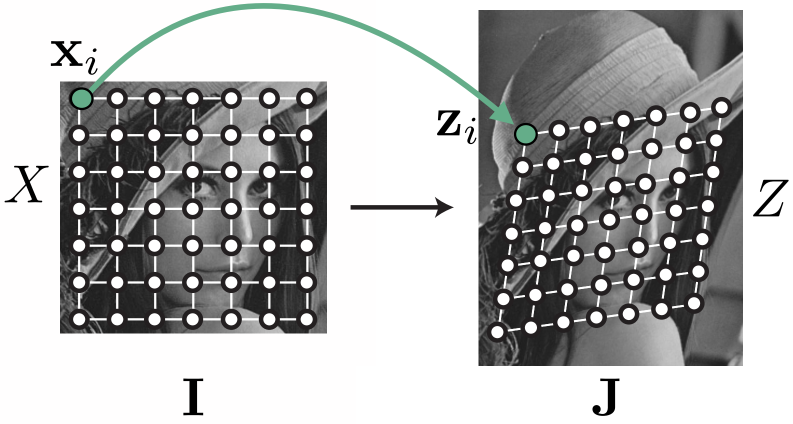


DEMO

2D Registration

...

2D Registration - Matching



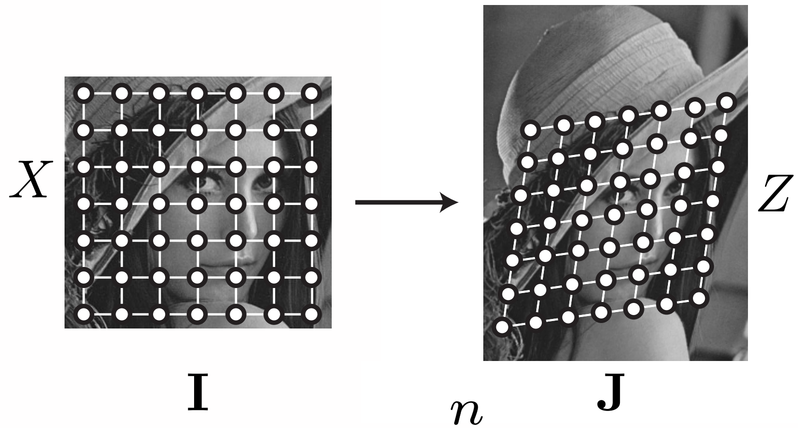
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

color value at $\mathbf{x}_i$ of image $\mathbf{I}$

color value at $\mathbf{z}_i$ of image $\mathbf{J}$

2D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

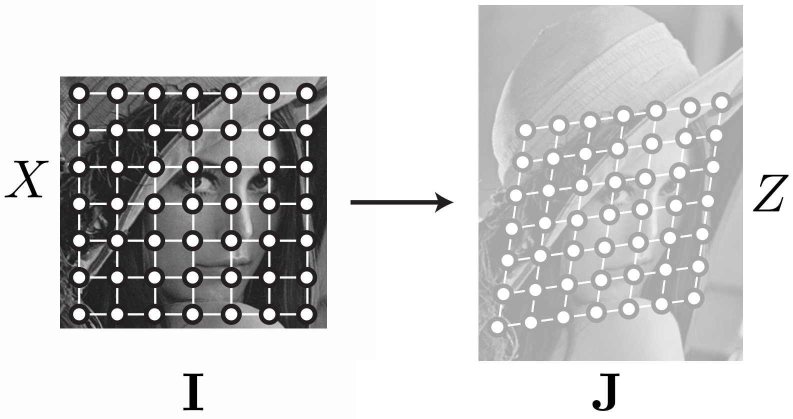
$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

$$\approx \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - [\mathbf{J}(\tilde{\mathbf{z}}_i) + \nabla\mathbf{J}(\tilde{\mathbf{z}}_i)^T(\mathbf{z}_i - \tilde{\mathbf{z}}_i)]\|_2^2$$

image gradient at $\tilde{\mathbf{z}}_i$ of image $\mathbf{J}$

initial guess
previous iteration

2D Registration - Matching



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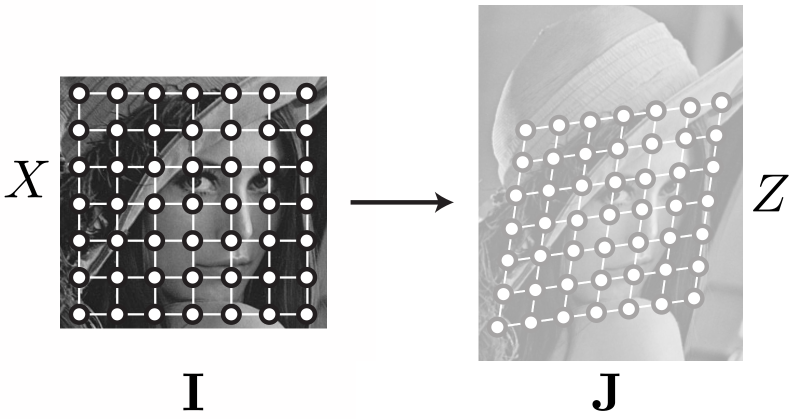
color value at $\mathbf{x}_i$ of image $\mathbf{I}$

color value at $\mathbf{z}_i$ of image $\mathbf{J}$

2D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\nabla \mathbf{I}(\mathbf{x}_i) - \nabla \mathbf{J}(\mathbf{z}_i)\|_2^2$$

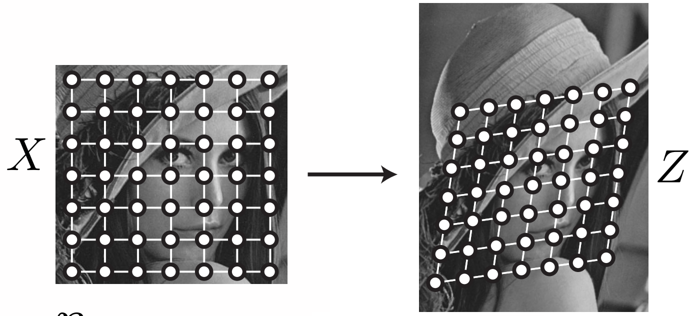
gradient at $\mathbf{x}_i$ of image $\mathbf{I}$

gradient at $\mathbf{z}_i$ of image $\mathbf{J}$

2D Registration - Prior

- Lucas-Kanade (1981)
  - Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \| (\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i) \|_2^2$$

"vertex" wants...

...its deformation...

...to match its neighbours'

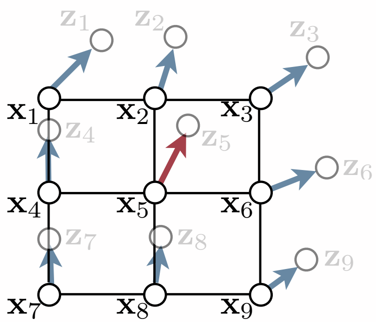
Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

2D Registration - Prior

- Lucas-Kanade (1981)
  - Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i)\|_2^2$$



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2D Registration - Prior

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$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

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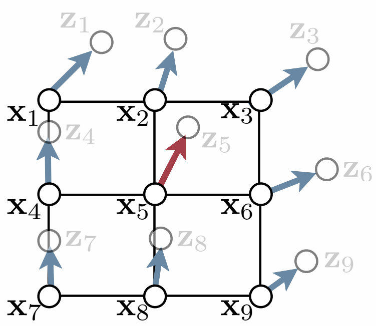

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2D Registration - Prior

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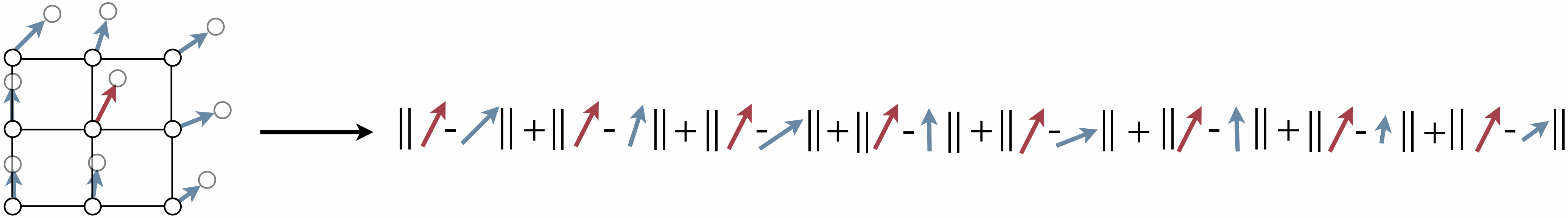
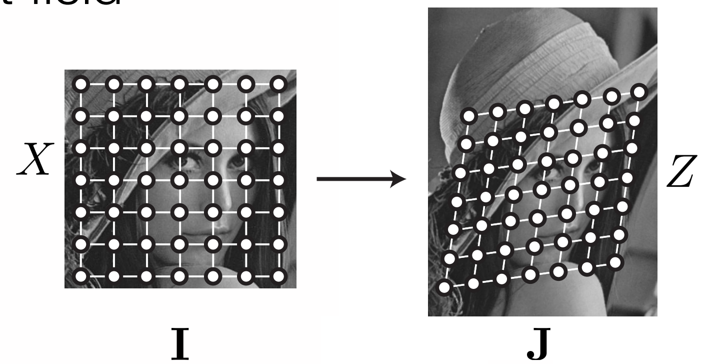


Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

2D Registration - Prior

- Horn-Schunck (1981)
  - Smooth displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



...to match its neighbours' average

$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \|(\mathbf{z}_i - \mathbf{x}_i) - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mathbf{z}_j - \mathbf{x}_j)\|_2^2$$

"vertex" wants...

...its deformation...

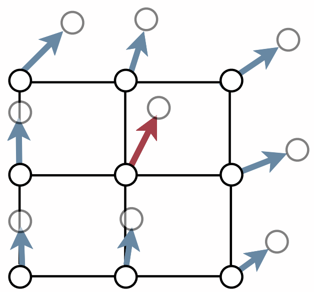
Horn, Schunk: "**Determining Optical Flow**", *Artificial Intelligence Journal 1981*

2D Registration - Prior

- Horn-Schunck (1981)
  - Smooth displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \|(\mathbf{z}_i - \mathbf{x}_i) - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mathbf{z}_j - \mathbf{x}_j)\|_2^2$$



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2D Registration - Prior

- Horn-Schunck (1981)
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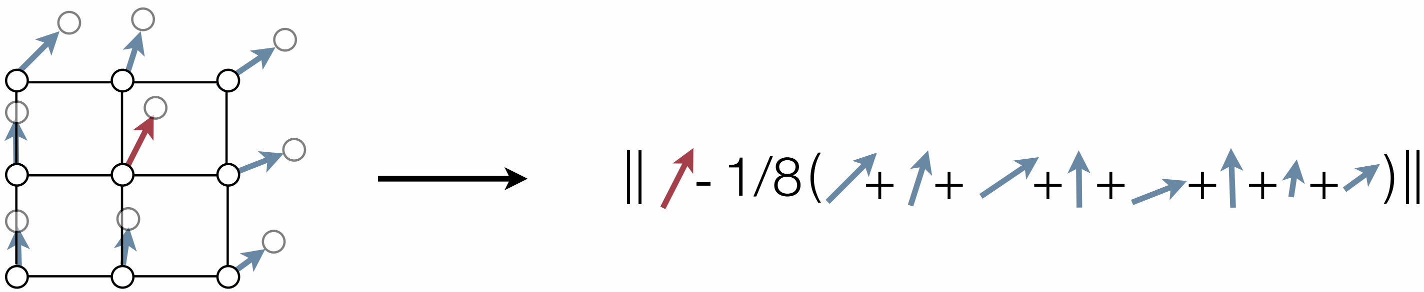
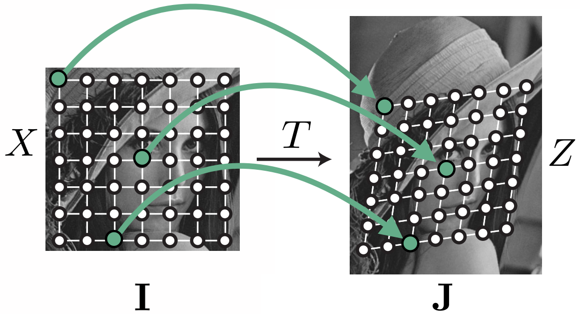


Horn, Schunk: “**Determining Optical Flow**”, *Artificial Intelligence Journal 1981*

2D Registration - Recap

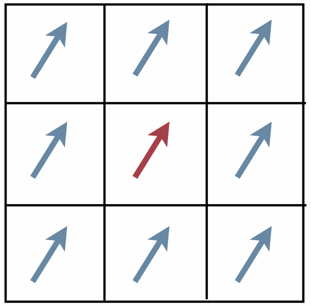
Matching



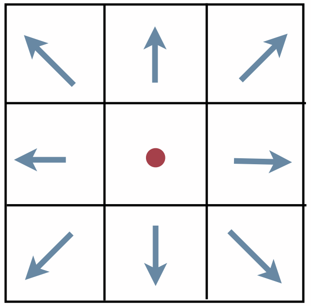
$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\nabla\mathbf{I}(\mathbf{x}_i) - \nabla\mathbf{J}(\mathbf{z}_i)\|_2^2$$

Prior



constant (Lucas-Kanade)



smooth (Horn-Schunck)

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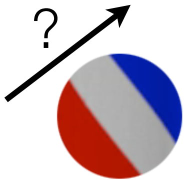
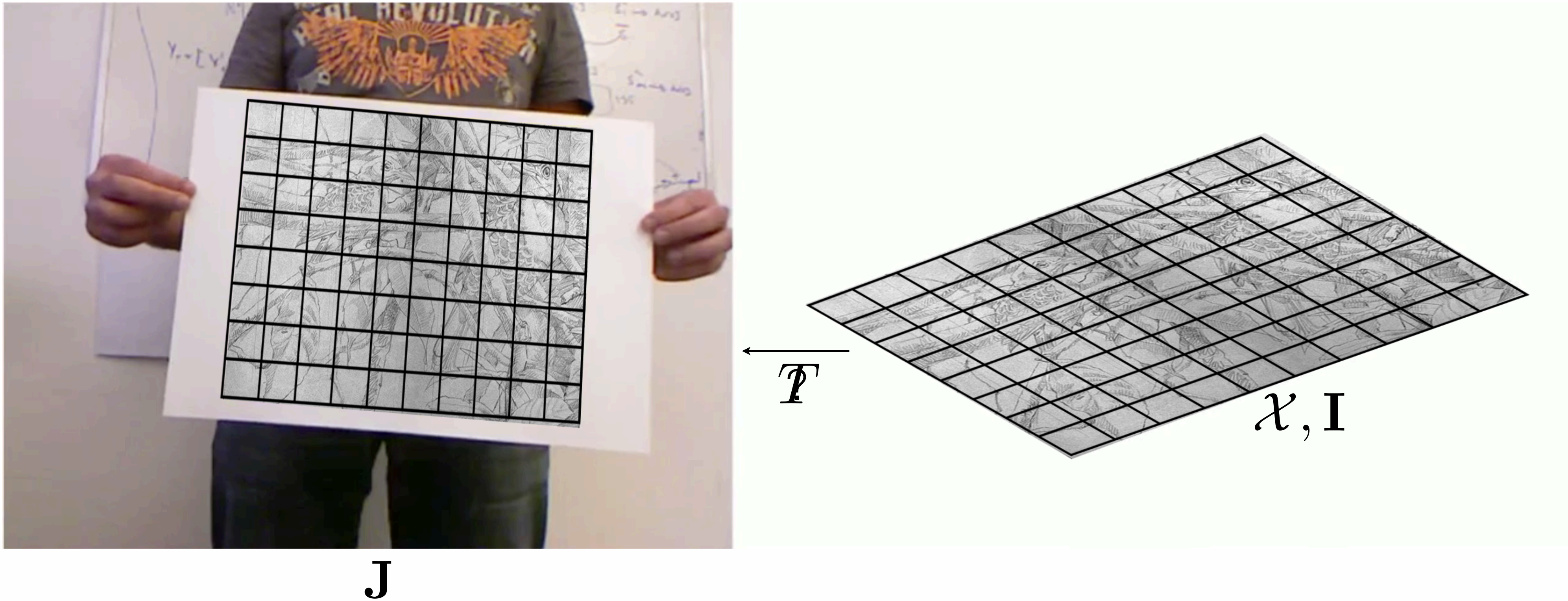
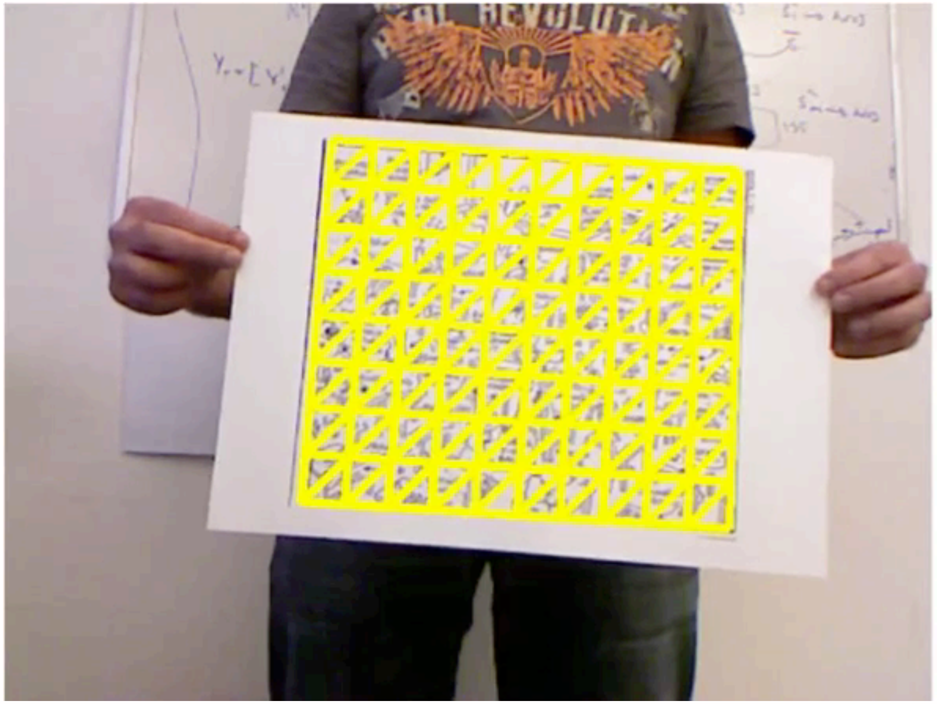
2D/3D Registration

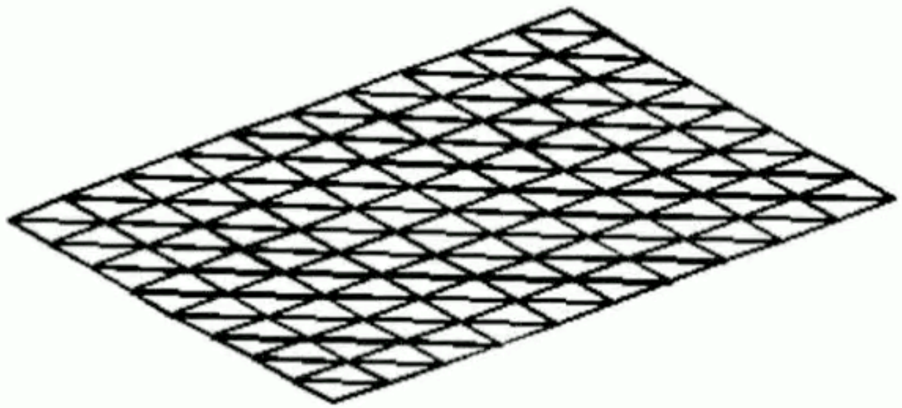


2D/3D Registration



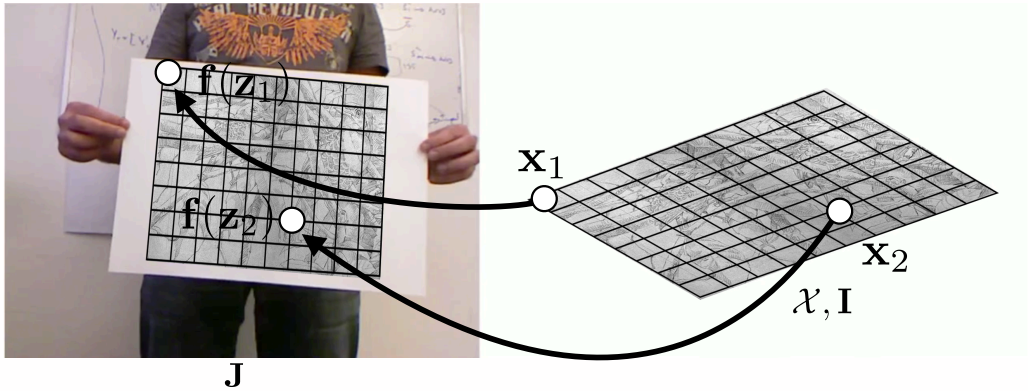
2D/3D Registration





[Ostlund et al. 2012]

2D/3D Registration

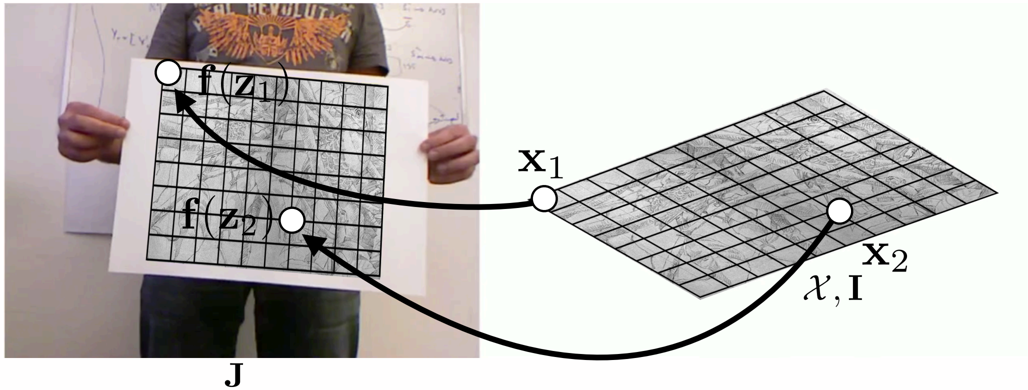


$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

projection of the 3D vertex
in the 2D image

2D/3D - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

Orthographic projection $$\mathbf{f}(\mathbf{z}_i) = \begin{bmatrix} \mathbf{z}_{i,x} & \mathbf{z}_{i,y} \end{bmatrix}^T$$

Perspective projection $$\mathbf{f}(\mathbf{z}_i) = \begin{bmatrix} \frac{f\mathbf{z}_{i,x}}{\mathbf{z}_{i,z}} & \frac{f\mathbf{z}_{i,y}}{\mathbf{z}_{i,z}} \end{bmatrix}^T$$

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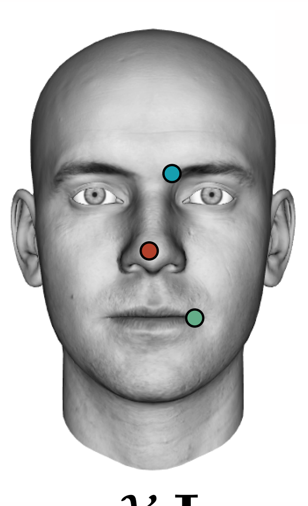
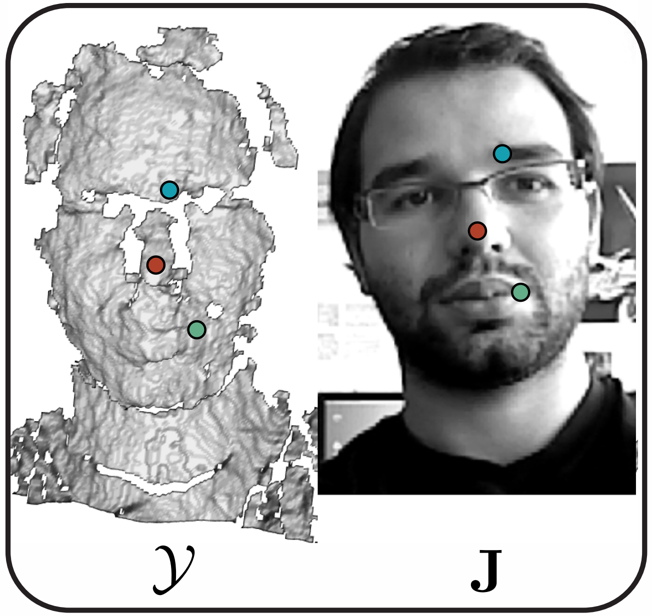
FÉDÉRALE DE LAUSANNE

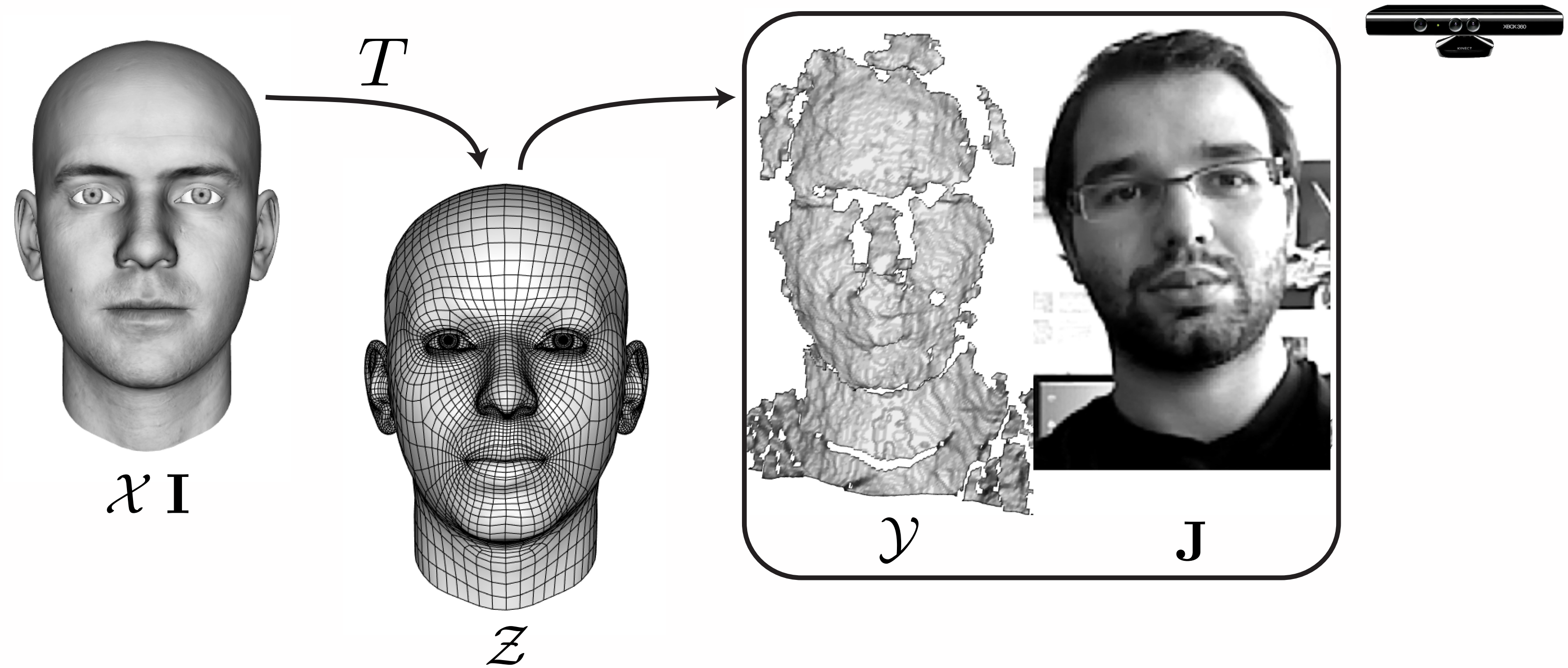
2D/3D Registration



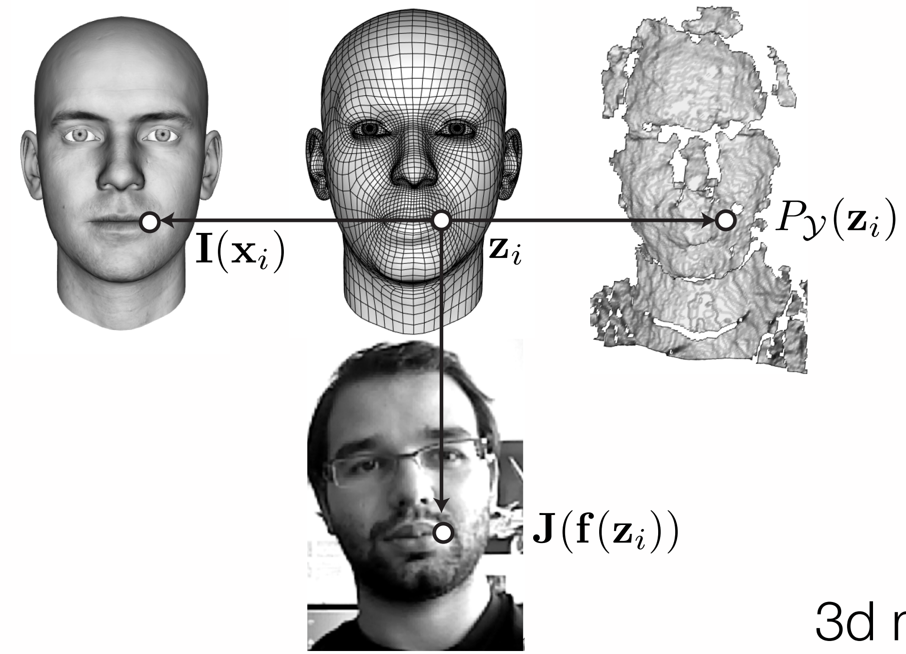
?



2D/3D Registration



2D/3D - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

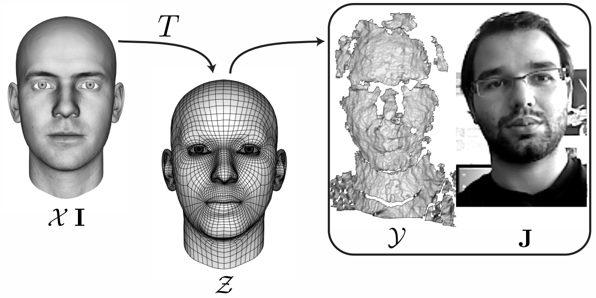
3d matching

2d matching

$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

2D/3D Registration - Recap

Matching



$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$
$$+ w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

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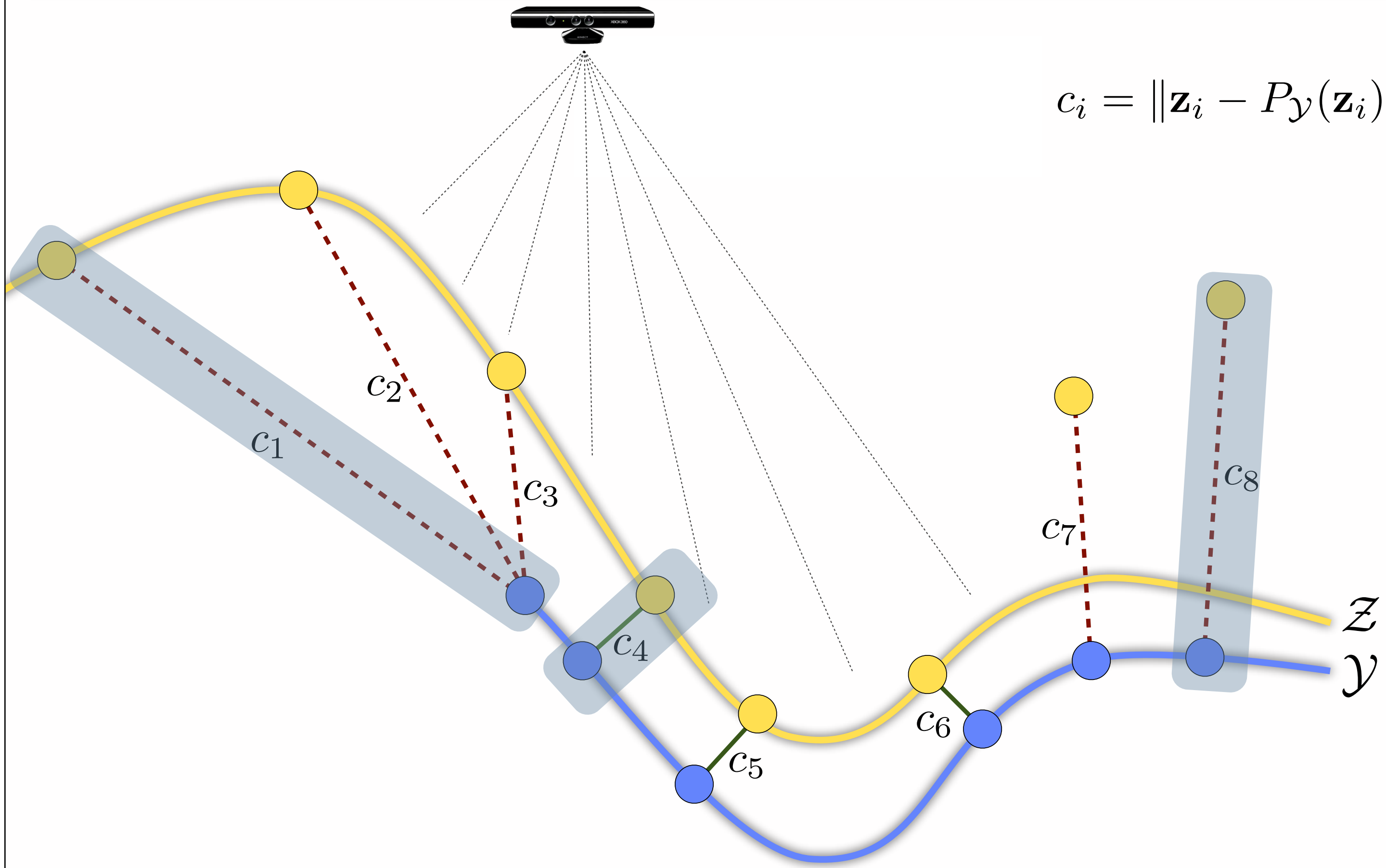
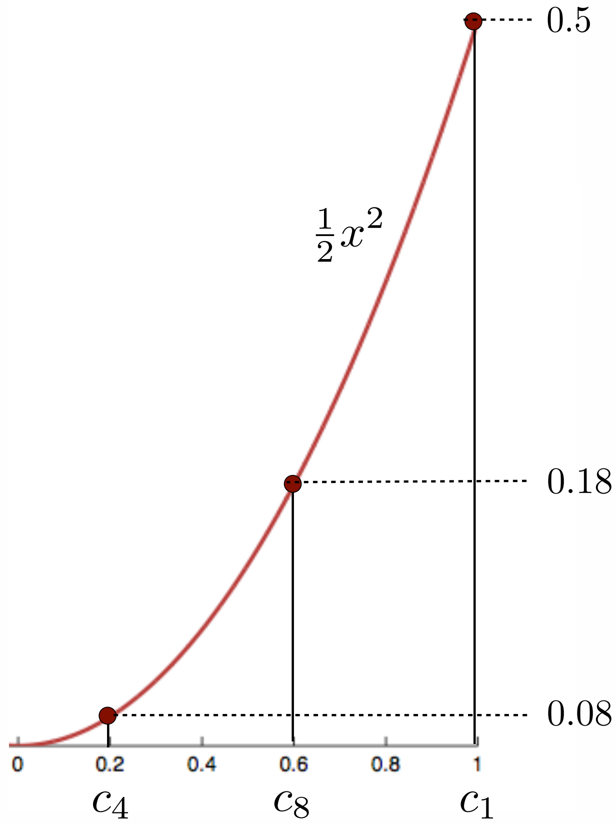
Outlook (5 min)

Robust Registration

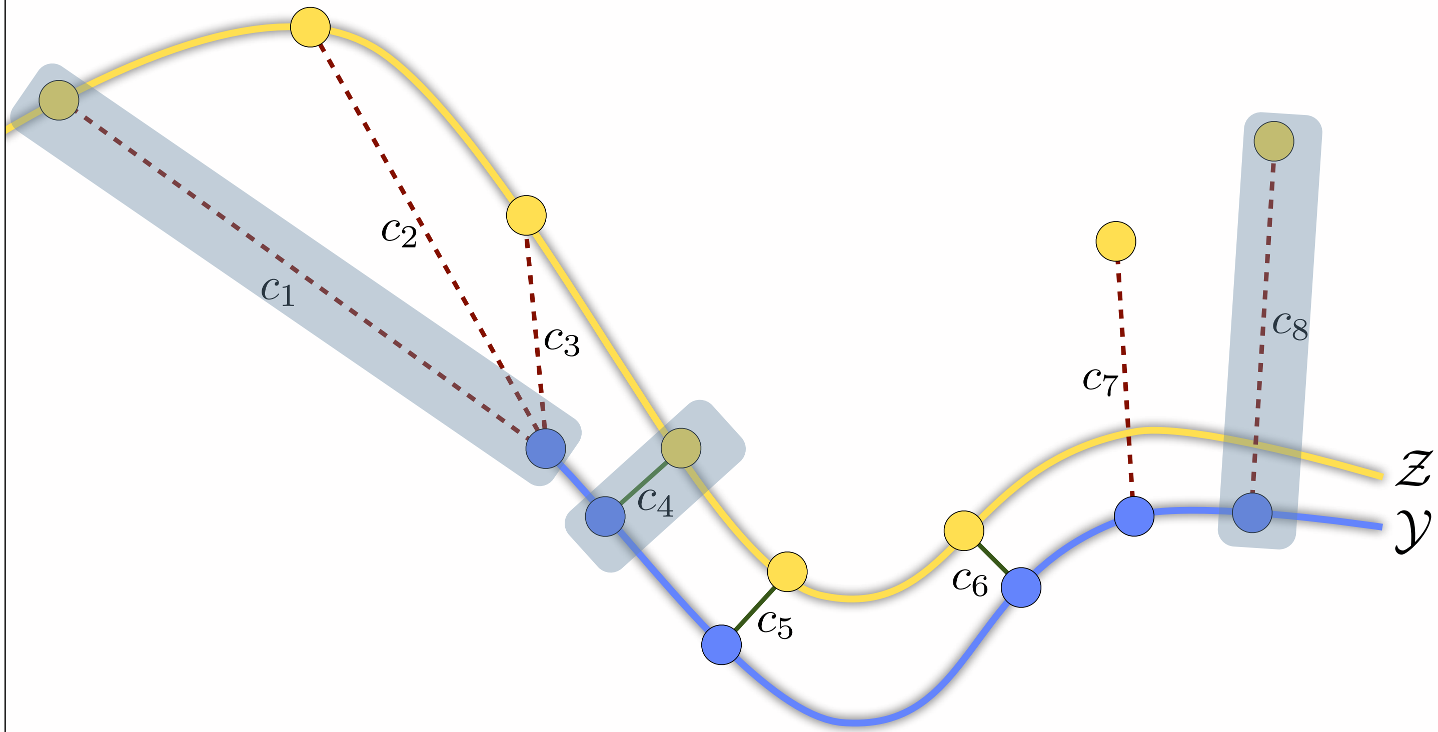


$$c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$

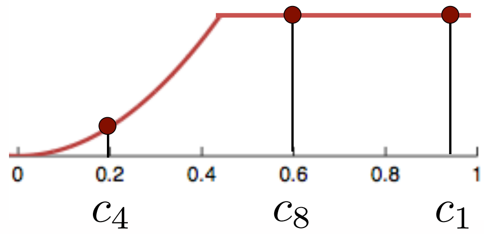


Robust Registration

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} c_i^2, \quad c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$

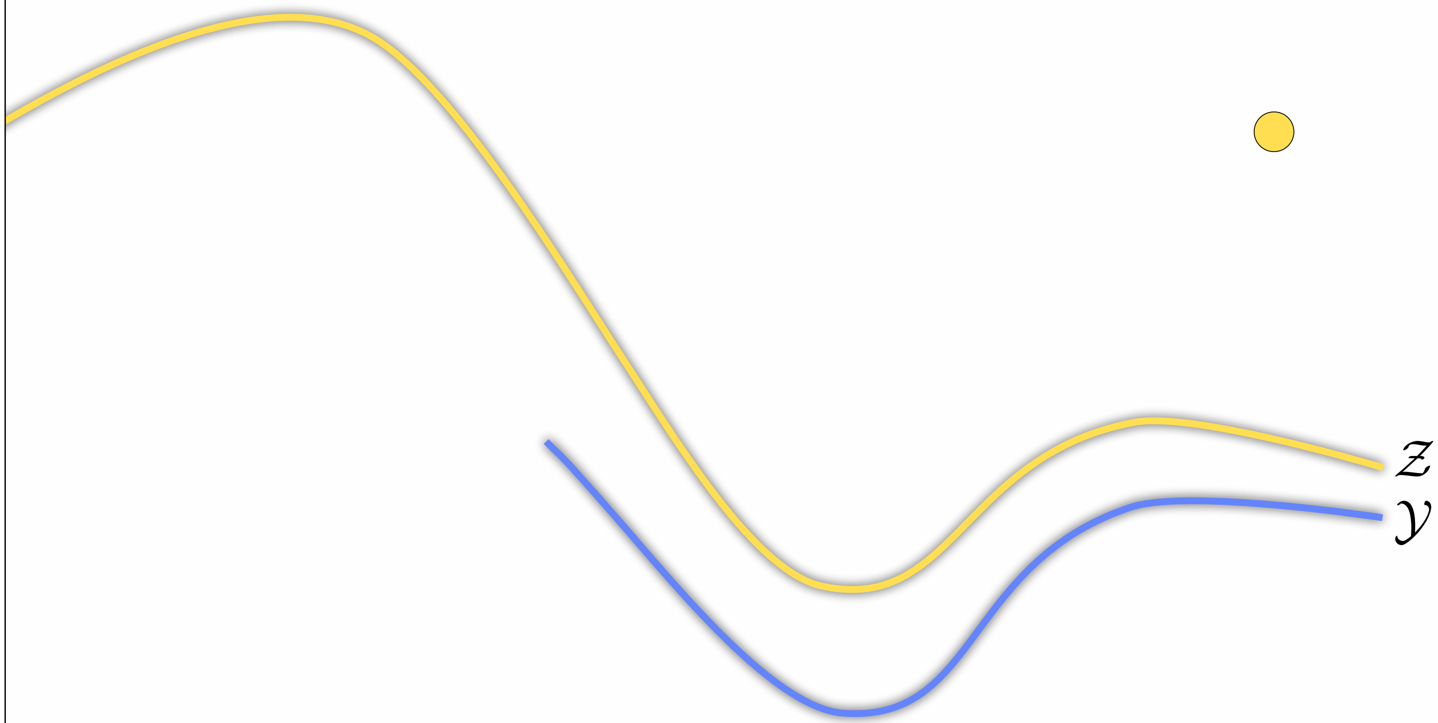


$$\begin{cases} \frac{1}{2}x^2 \text{ if } |x| \leqslant \tau \\ \frac{1}{2}\tau^2 \text{ otherwise} \end{cases}$$

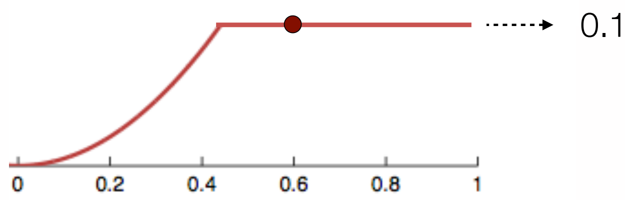


Robust Registration

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} c_i^2, \quad c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$



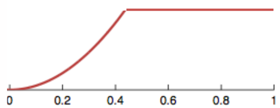
$$E_{\text{match}}(\mathbf{z}) = cst$$
$$\nabla E_{\text{match}}(\mathbf{z}) = 0$$



Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \Longrightarrow \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

robust function



IRLS

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

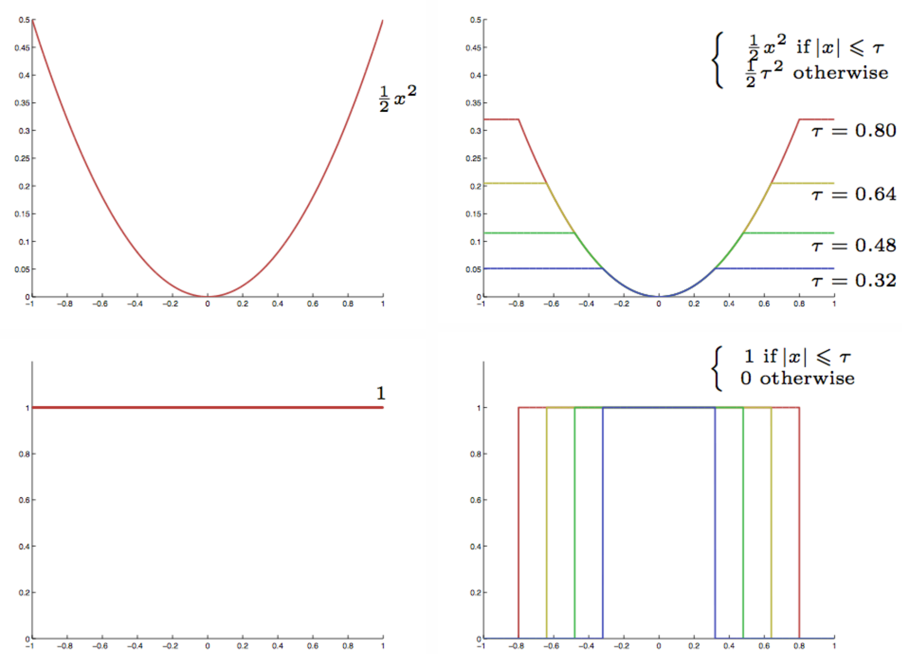
weight function   previous iteration residuals

Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

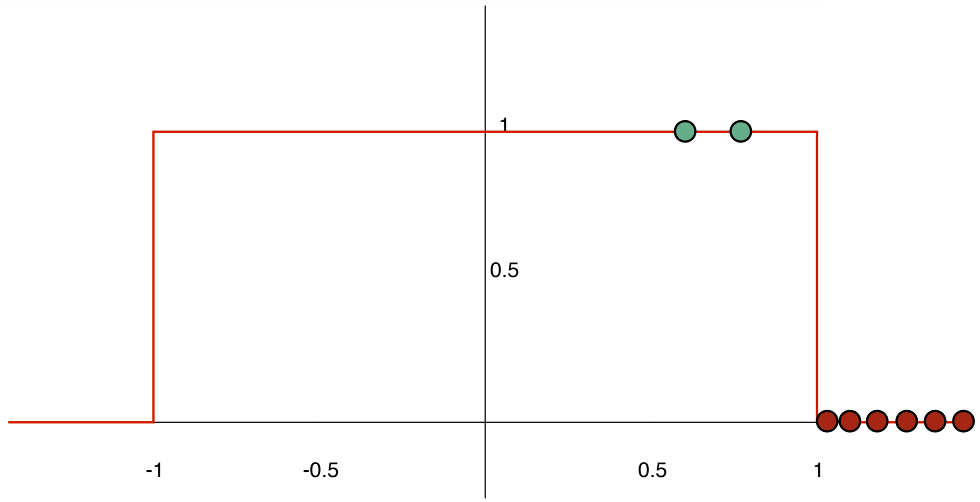
$\varphi(x)$

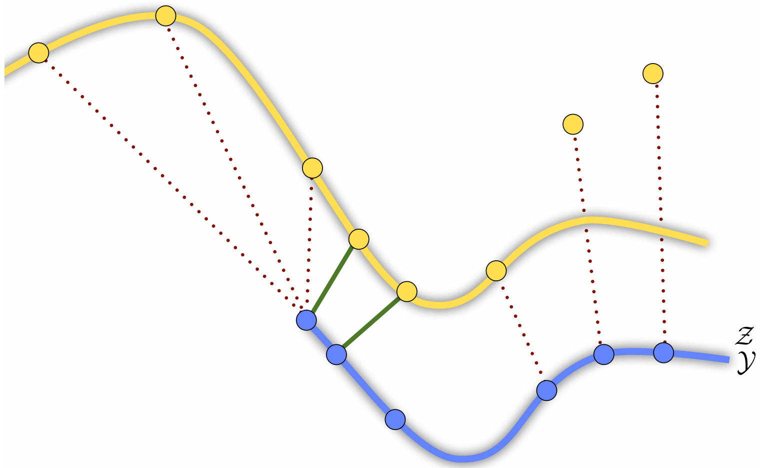
$w(x)$



Robust Registration

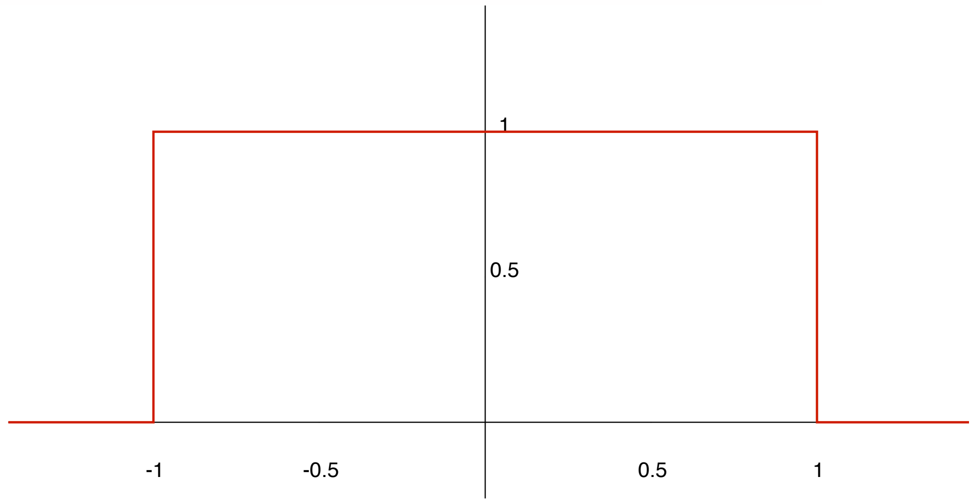
$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

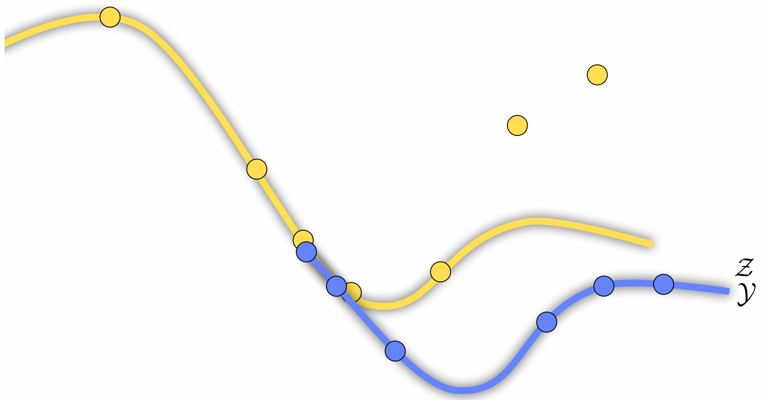




Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

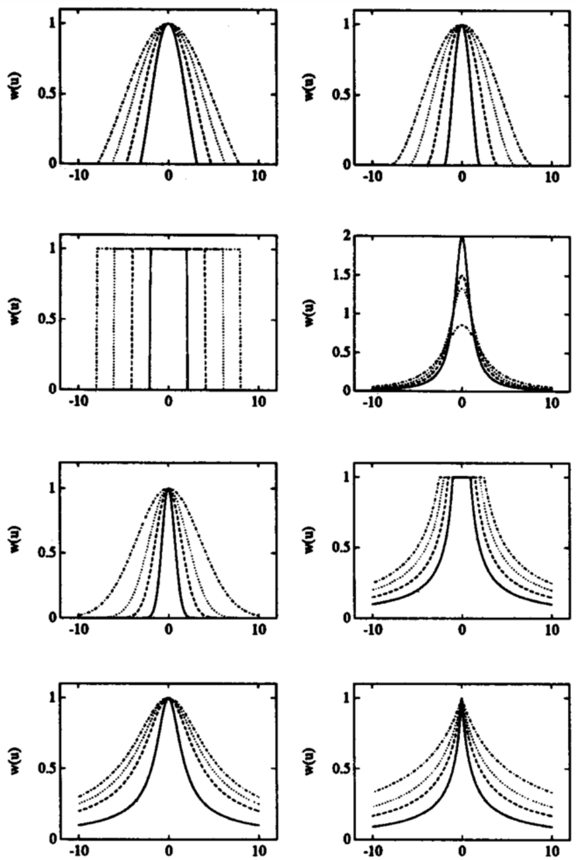




Robust Registration

TABLE I
WEIGHT FUNCTION

Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments
Andrews' wave (a)	$a^2[1-\cos(u/a)]$ $2a^2$	$a\sin(u/a)$ 0	$(u/a)^{-1}\sin(u/a)$ 0	$\|u\| \leq \pi a$ $\|u\| > \pi a$	$1.0 \leq a \leq 2.5$
Tukey's Biweight (b)	$(b^2/2)[1-(1-(u/b)^2)]^3$ $b^2/2$	$u[1-(u/b)^2]^2$ 0	$[1-(u/b)^2]^2$ 0	$\|u\| \leq b$ $\|u\| > b$	$2.0 \leq b \leq 8.0$
Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\|u\| \leq t$ $\|u\| > t$	$2.0 \leq t \leq 8.0$
Student-t(f)			$(1+f)/(f+u^2)$		Derived in [1] by ML analysis
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \leq w \leq 5.0$
Huber(h)	$u^2/2$ $h\|u\| - h^2/2$	u h sig(u)	1 $h\|u\|^{-1}$	$\|u\| \leq h$ $\|u\| > h$	$1.0 \leq h \leq 2.5$
Logistic(l)	$l^2\log[\cosh(u/l)]$	$l\tanh(u/l)$	$(u/l)^{-1}\tanh(u/l)$		$1.0 \leq l \leq 3.0$
Fair (f)	$f^2[\|u\|/f - \log(1+\|u\|/f)]$	$u(1+\|u\|/f)^{-1}$	$(1+\|u\|/f)^{-1}$		$1.0 \leq f \leq 5.0$



Mirza, Boyer: **Performance Evaluation of a Class of M-Estimators for Surface Parameter Estimation in Noisy Range Data,** *IEEE Transactions on Robotics and Automation*

Robust Registration

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Student-t(f)			$(1+f)/(f$		
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \leq w \leq 5.0$
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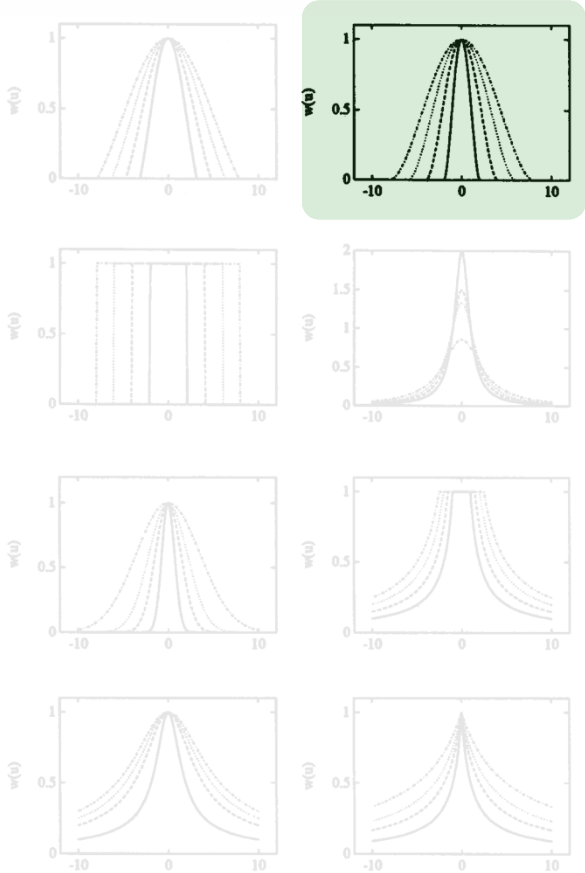
DEMO



Mirza, Boyer: **Performance Evaluation of a Class of M-Estimators for Surface Parameter Estimation in Noisy Range Data,** *IEEE Transactions on Robotics and Automation 1993*

Robust Registration

TABLE I
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Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments	(a)
Andrews' wave (a)	$a^2[1 - \cos(u/a)]$ $2a^2$	$a \sin(u/a)$ 0	$(u/a)^{-1} \sin(u/a)$ 0	$\|u\| \le \pi a$ $\|u\| > \pi a$	$1.0 \le a \le 2.5$	(b)
Tukey's Biweight (b)	$(b^2/2)[1 - (1 - (u/b)^2)]^3$ $b^2/2$	$u[1 - (u/b)^2]^2$ 0	$[1 - (u/b)^2]^2$ 0	$\|u\| \le b$ $\|u\| > b$	$2.0 \le b \le 8.0$	(c)
Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\|u\| \le t$ $\|u\| > t$	$2.0 \le t \le 8.0$	
Student-t(f)			$(1 + f)/(f + u^2)$		Derived in [1] by ML analysis	(d)
Cauchy(c)	$(c^2/2)\log[1 + (u/c)^2]$	$u[1 + (u/c)^2]^{-1}$	$[1 + (u/c)^2]^{-1}$		special case of t-distribution	
Welsch(w)	$(w^2/2)[1 - \exp[-(u/w)^2]]$	$u \exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$	(e)
Huber(h)	$u^2/2$ $h\|u\| - h^2/2$	u $h \operatorname{sig}(u)$	1 $h\|u\|^{-1}$	$\|u\| \le h$ $\|u\| > h$	$1.0 \le h \le 2.5$	(f)
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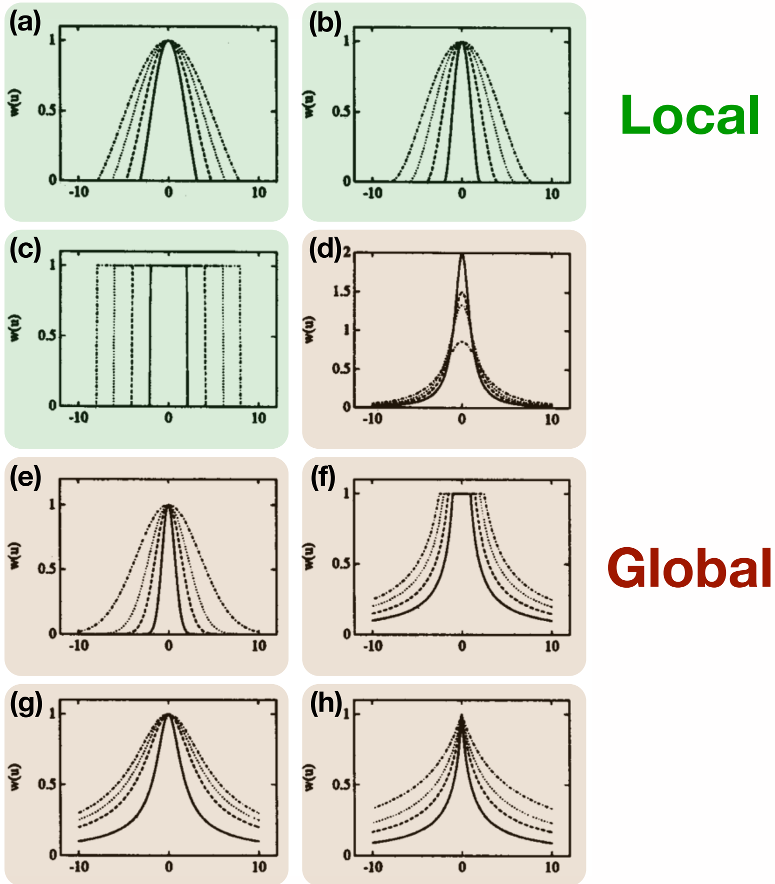
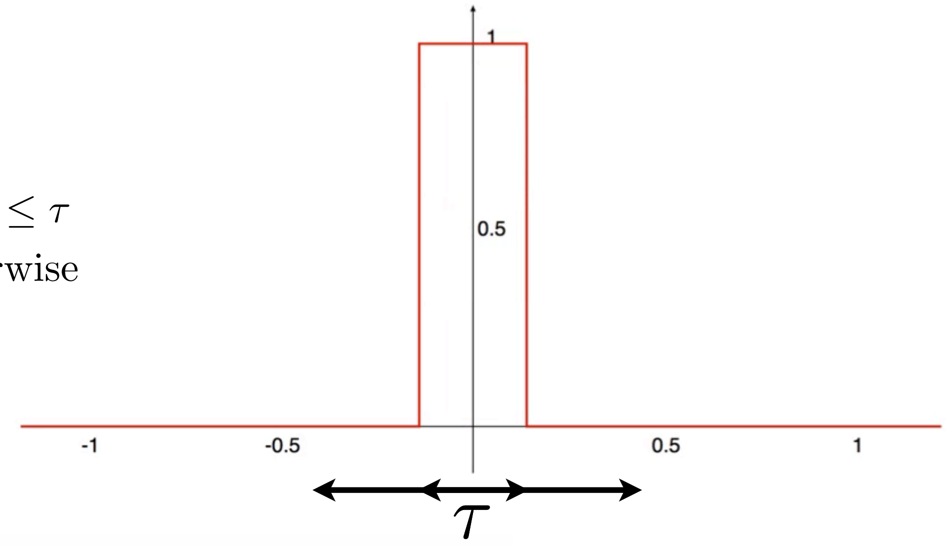


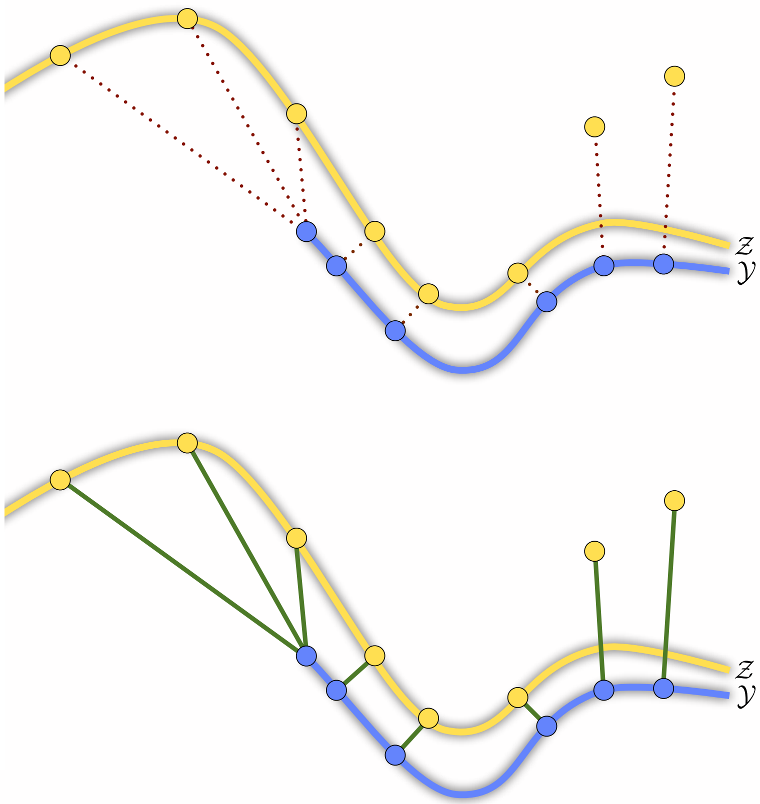
Mirza, Boyer: **Performance Evaluation of a Class of M-Estimators for Surface Parameter Estimation in Noisy Range Data,** *IEEE Transactions on Robotics and Automation 1993*

Robust Registration

- Local Support

$$w(x) = \begin{cases} 1 & \text{if } |x| \leq \tau \\ 0 & \text{otherwise} \end{cases}$$

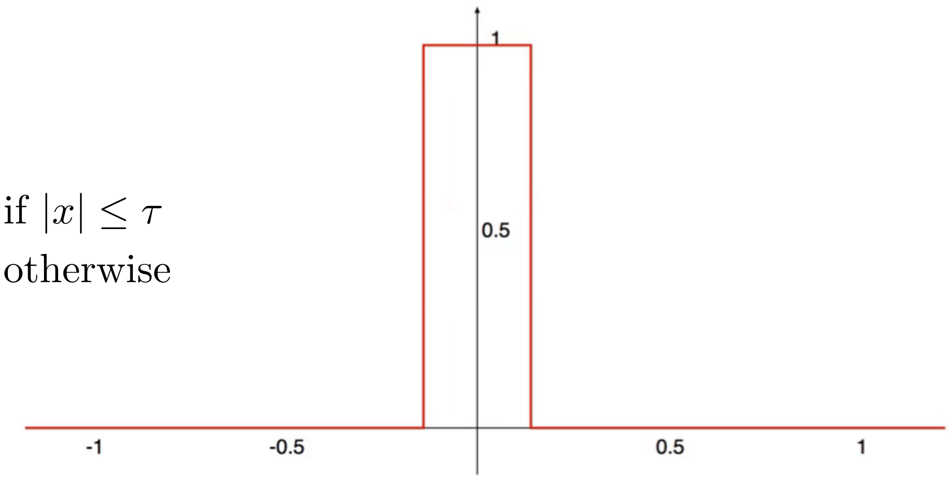




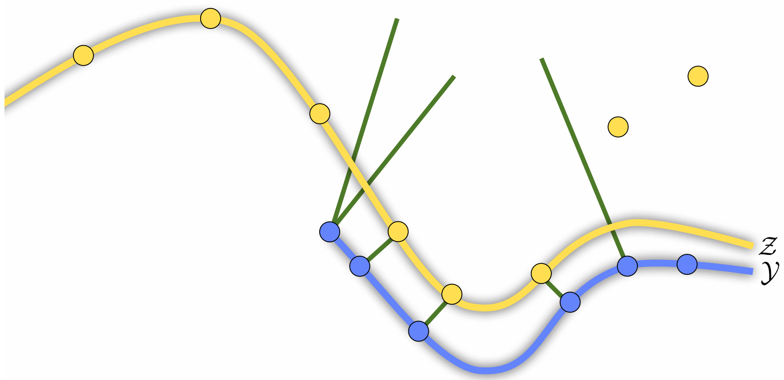
Robust Registration

- Trimmed Metrics
  - *Known number of inlier N*

$$w(x) = \begin{cases} 1 & \text{if } |x| \le \tau \\ 0 & \text{otherwise} \end{cases}$$



N=3



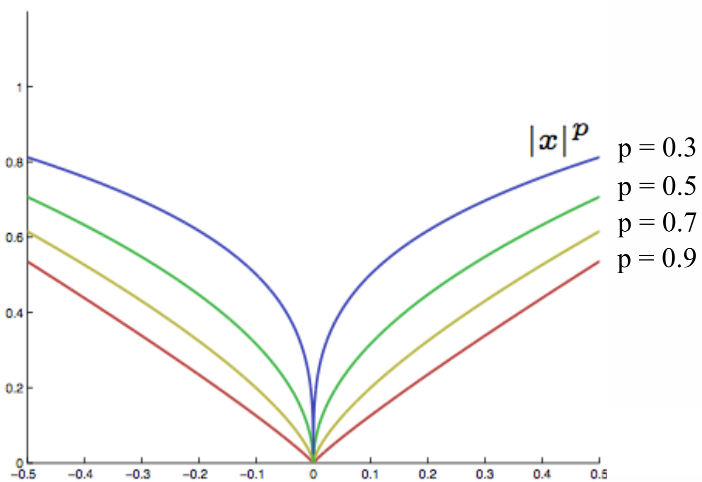
Chetverikov, Svirko, Stepanov, Krsek
The Trimmed Iterative Closest Point Algorithm,
International Conference on Pattern Recognition 2002

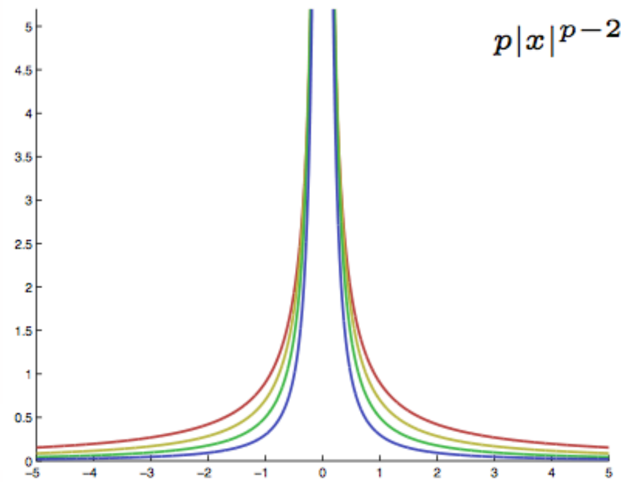
Robust Registration

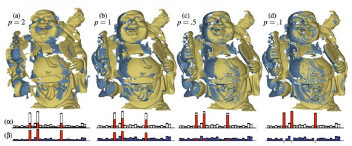
- Sparse Metrics

$$\varphi(x) = |x|^p$$

$$w(x) = p|x|^{p-2}$$





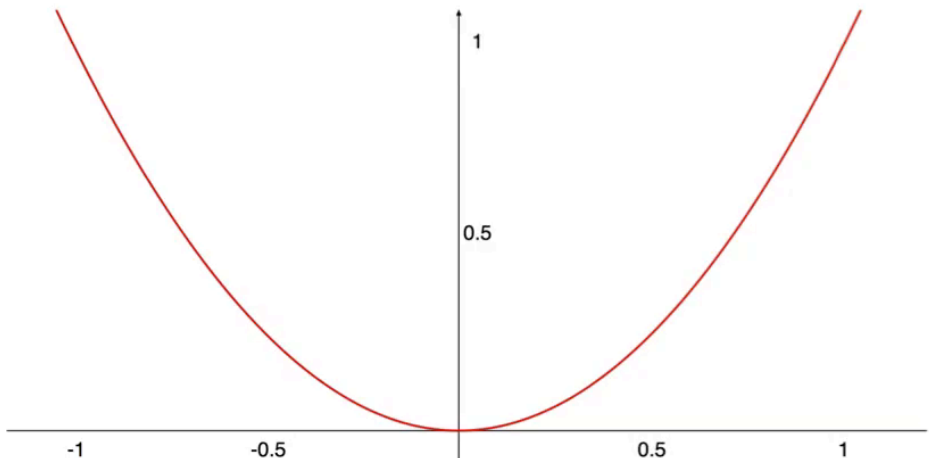


Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
SGP 2013

Robust Registration

- Sparse Metrics

$$\varphi(x) = |x|^p$$



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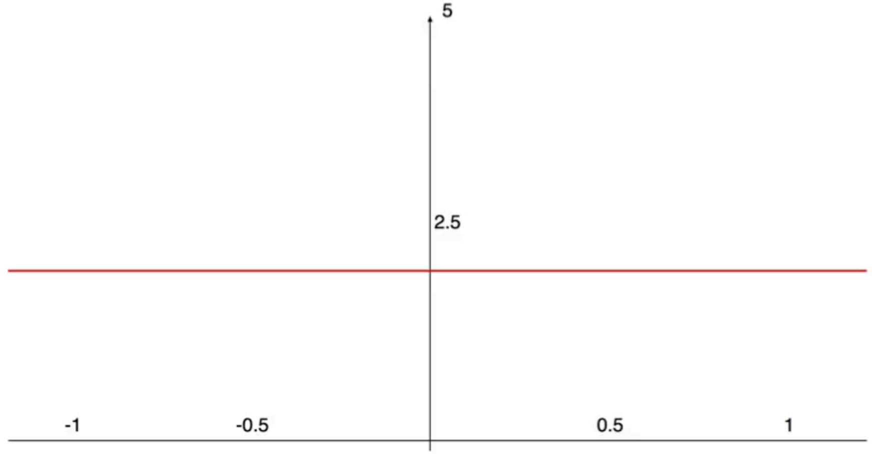
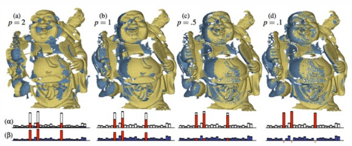
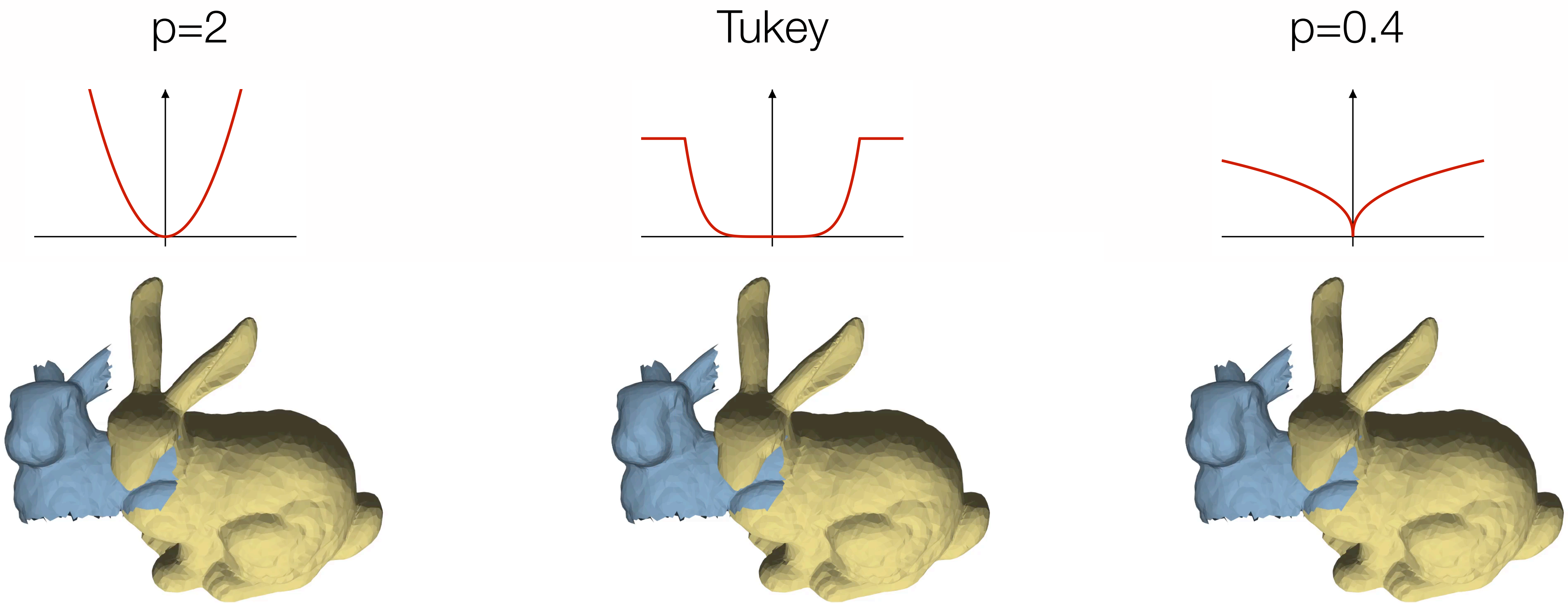
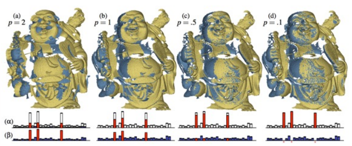




Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
SGP 2013

Robust Registration





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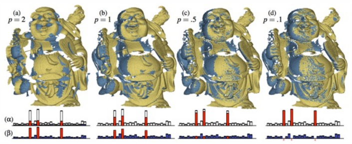
SGP 2013

Robust Registration

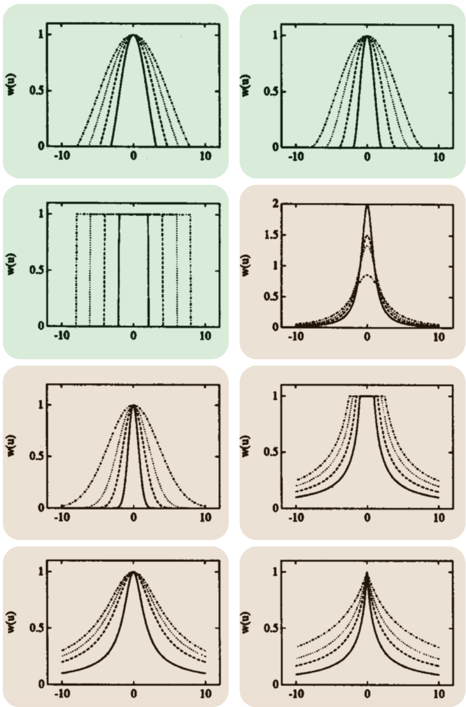


Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
SGP 2013

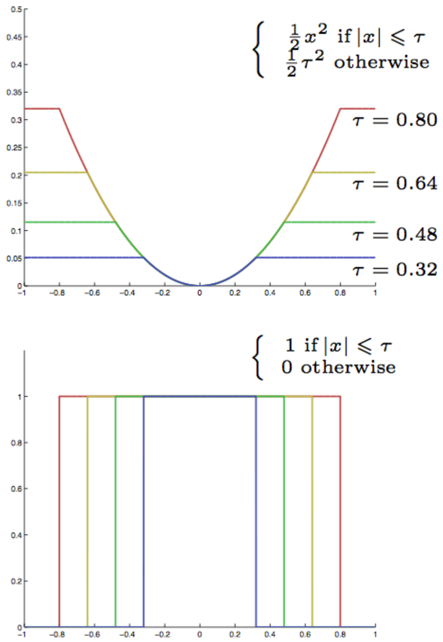
https://github.com/opengp/sparseicp

Robust Registration - Recap

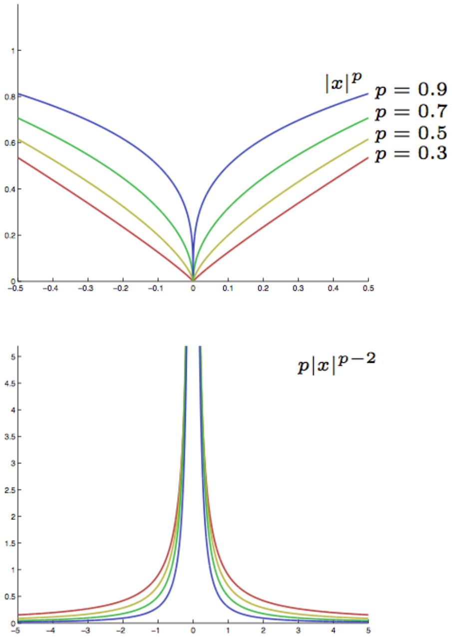
Local/Global



Trimmed



Sparse



ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Overview

Introduction (5min)

- Registration (5 min)
  - 3D Geometry (25 min)
  - 2D Images (10 min)
  - Combined 2D/3D (5 min)
  - Robust Registration (15 min)
  -  Q&A (5min)

- Applications
  - Rigid Scanning (10 min)
  - Articulated Tracking (10 min)
  - Non-rigid Modeling (10 min)
  - Realtime Face Tracking (10 min)
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Outlook (5 min)

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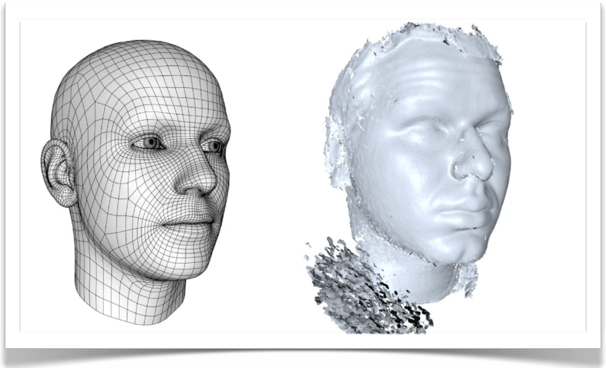
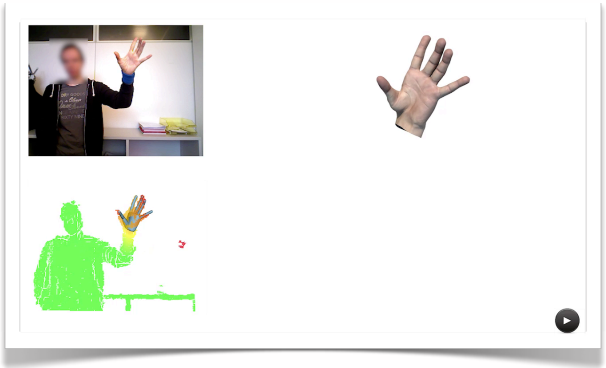
Applications

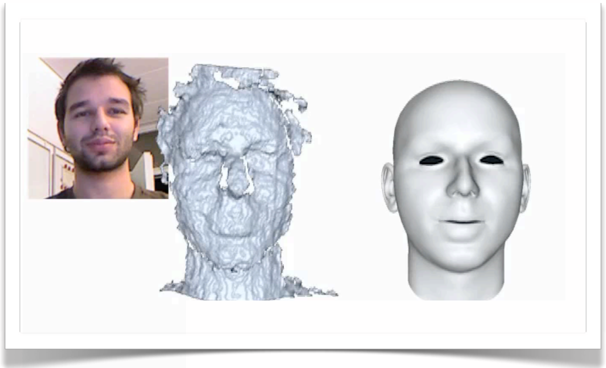
Rigid Scanning



Non-Rigid Modeling

Articulated Tracking





Face Tracking

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FÉDÉRALE DE LAUSANNE

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Outlook (5 min)

Rigid Scanning

KSCAN3D

ImFusion

Artec Group
3D Scanning Technologies

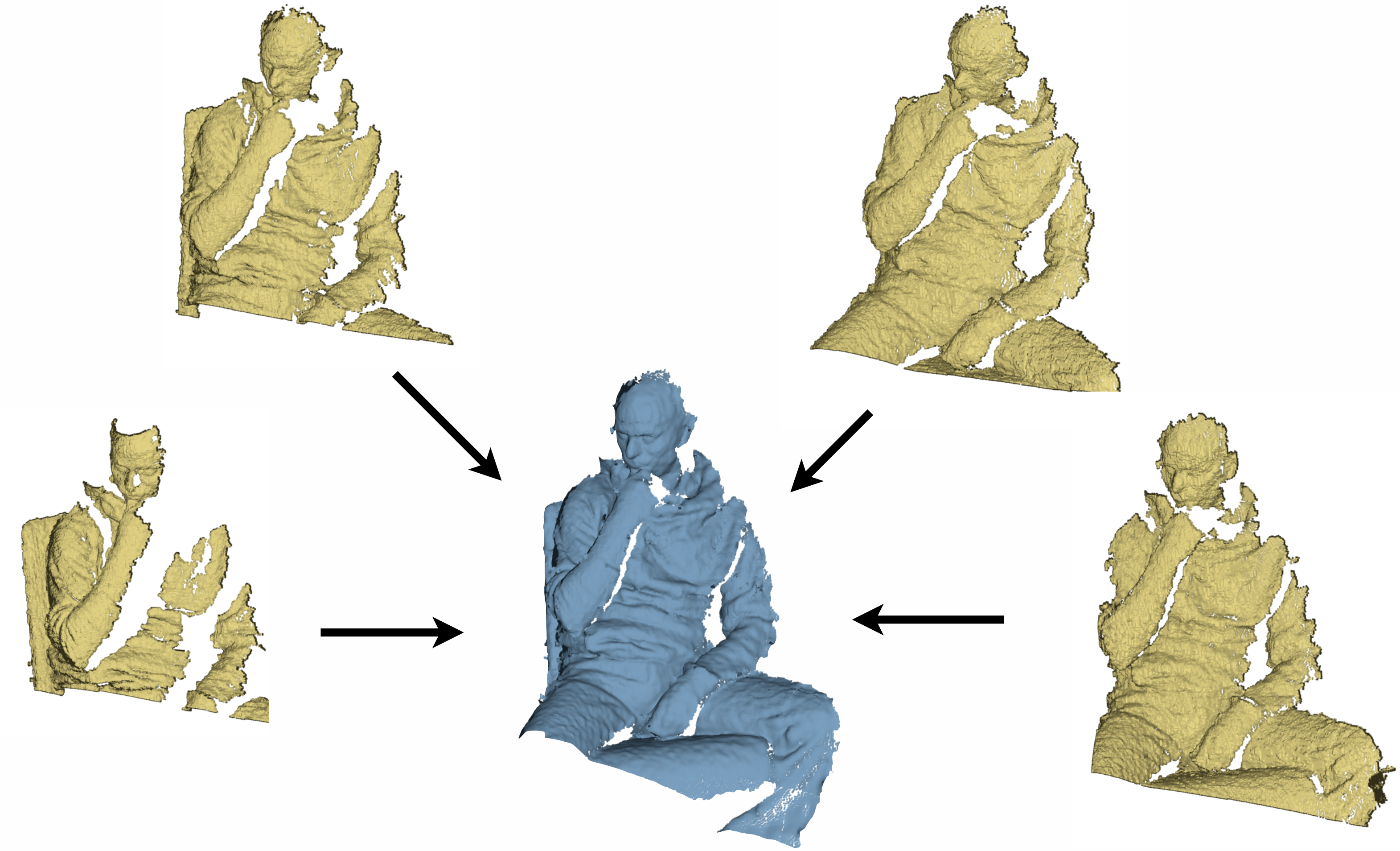
PROFACTOR®

FARO®

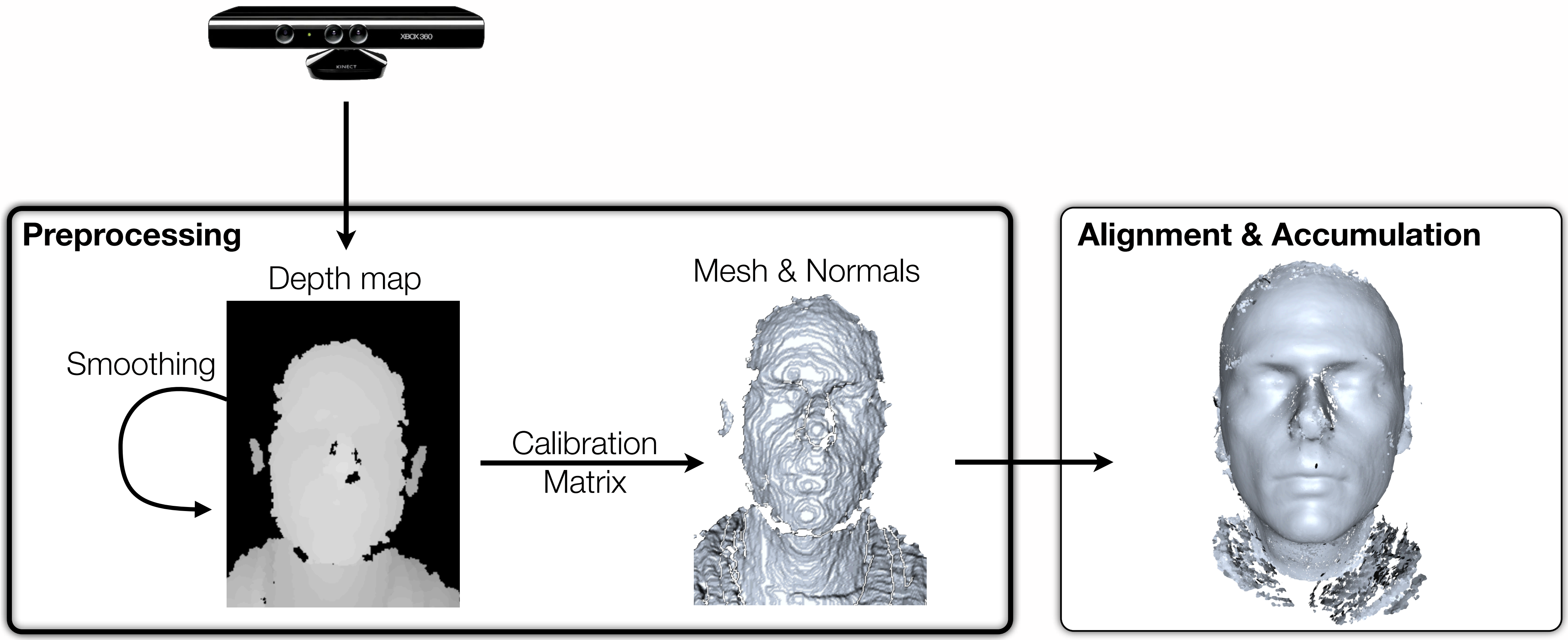
Microsoft®

SKANECT

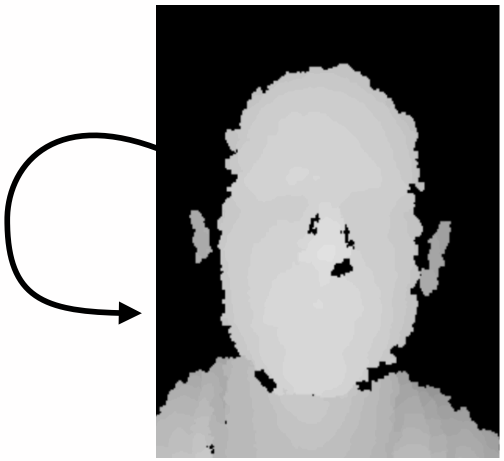
Rigid Scanning

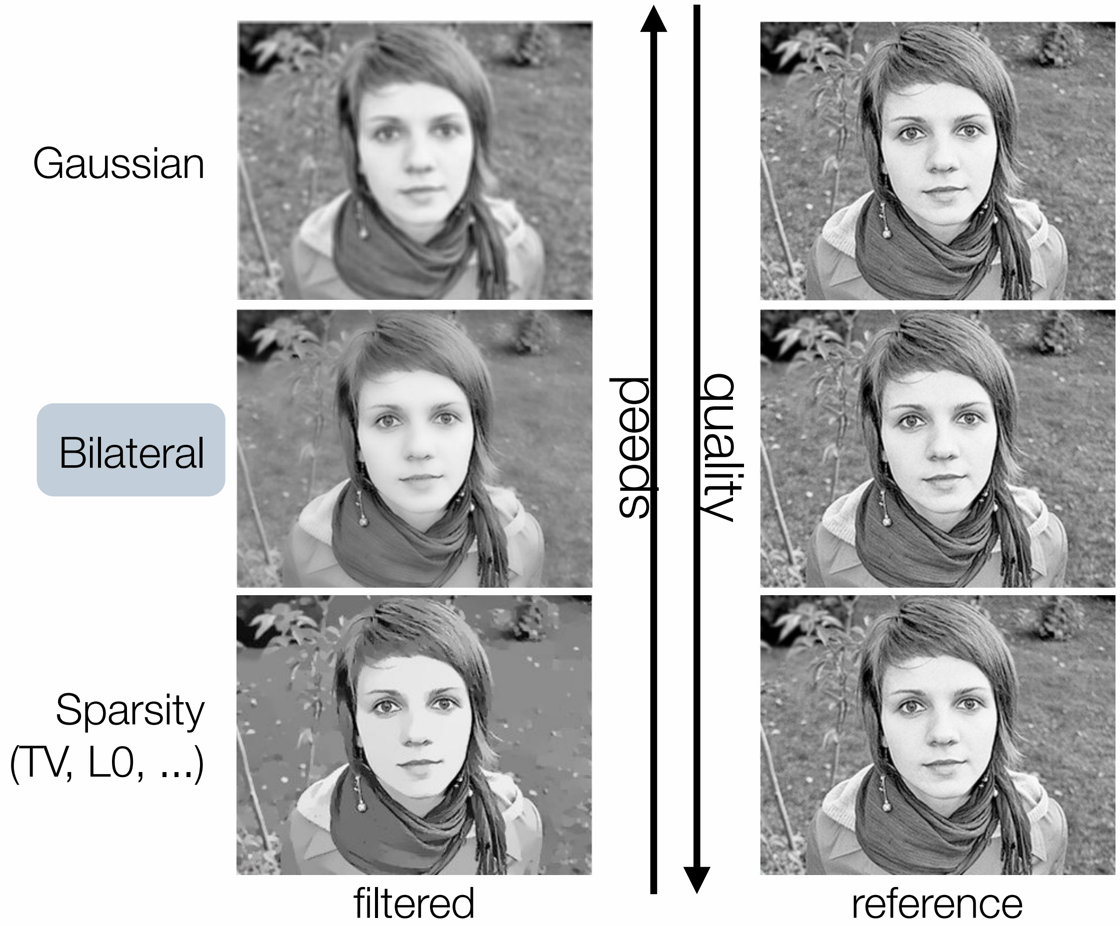


Rigid Scanning - Pipeline



Rigid Scanning - Preprocessing





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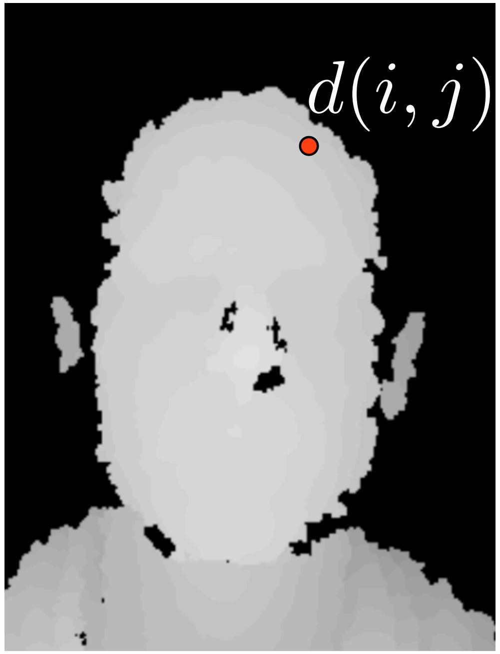
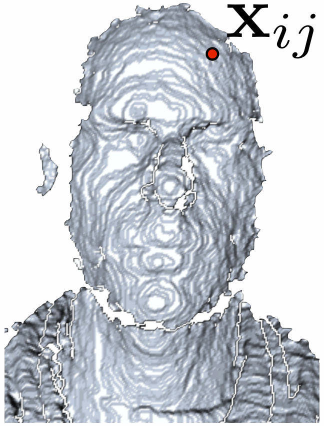


vertex positions



$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d(i,j) \\ j \cdot d(i,j) \\ d(i,j) \end{bmatrix}$$

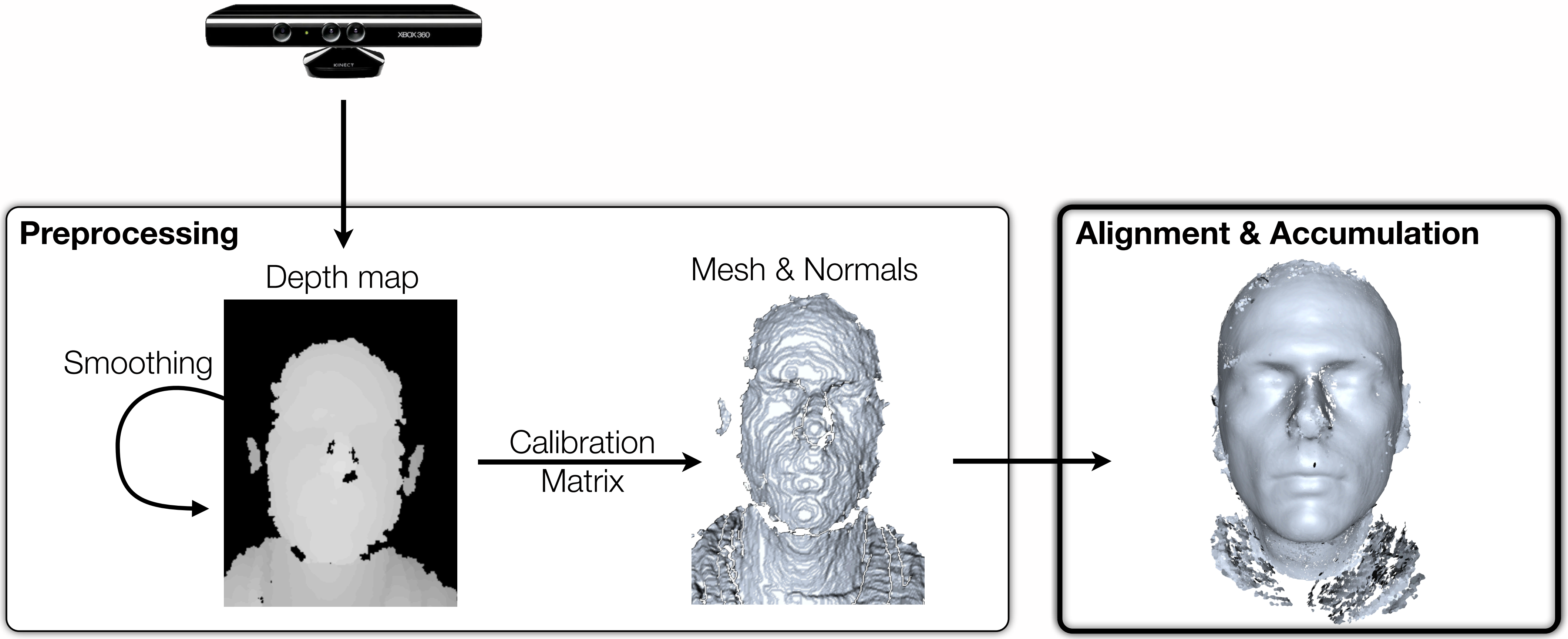
camera intrinsics (device dependent)

inverse of "persp. division"

vertex normals

finite differences (grid)

Rigid Scanning - Pipeline



Rigid Scanning - Optimization

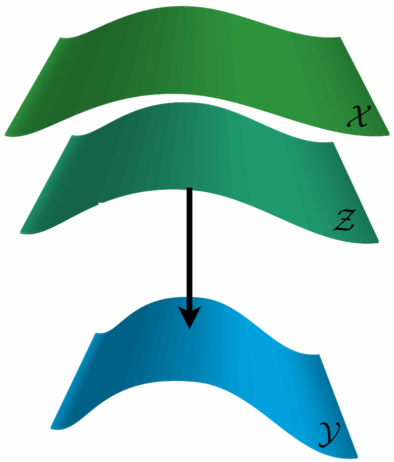
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

matching term

rigid motion

$\Downarrow \; w_2 \rightarrow \infty$

$$\arg\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \|(\mathbf{R}\mathbf{x}_i + \mathbf{t}) - P_{\mathcal{Y}}(\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



$\Downarrow$ traditional ICP

Step 1: find correspondences

Step 2: find best rigid transformation

Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$

$$\arg\min_{Z^{t+1},\tilde{\mathbf{R}},\tilde{\mathbf{t}}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^{t})\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t\mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

use previous iteration

Rigid Scanning - Optimization

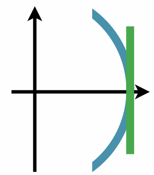
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$

$$\arg\min_{Z^{t+1},\tilde{\mathbf{R}},\tilde{\mathbf{t}}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t\mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

linearize rotation matrix

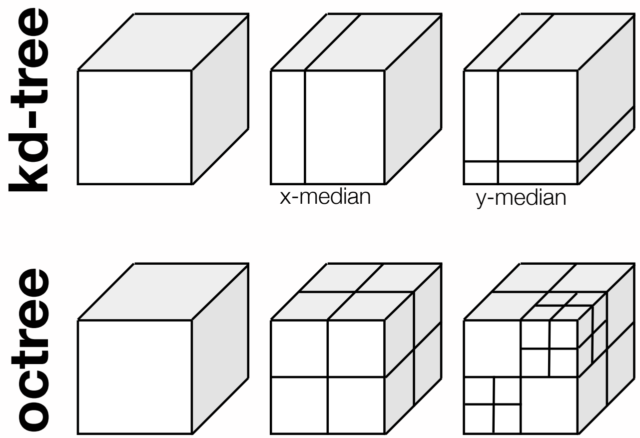
$$\mathbf{R} \approx \tilde{\mathbf{R}} = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix}$$



Rigid Scanning - Optimization

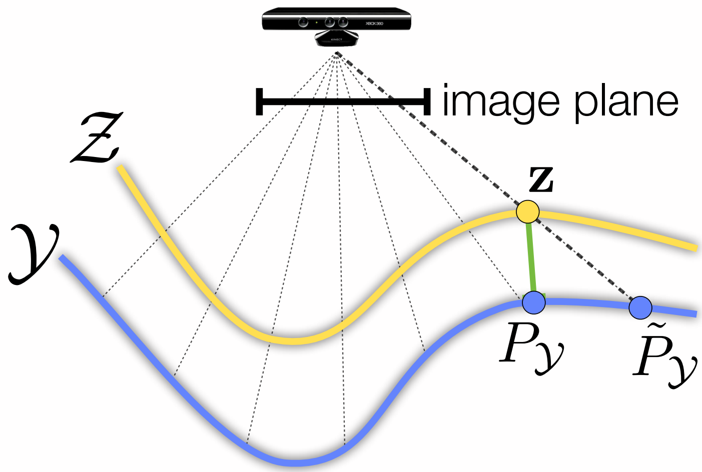
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Acceleration Data Structures



build: $O(n \log n)$, each query: $O(\log n)$

Projective Lookup

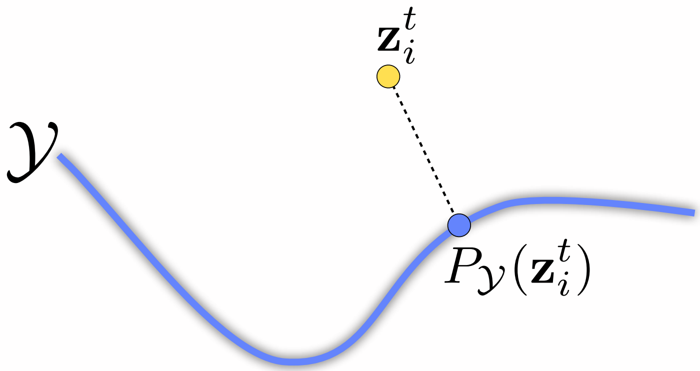


build: $O(1)$, each query: $O(1)$

Rigid Scanning - Optimization

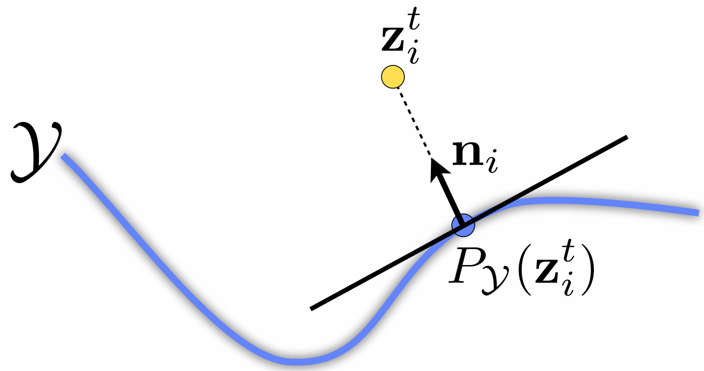
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Point-to-Point



$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$

Point-to-Plane



$$\langle \mathbf{n}_i, \mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t) \rangle^2$$

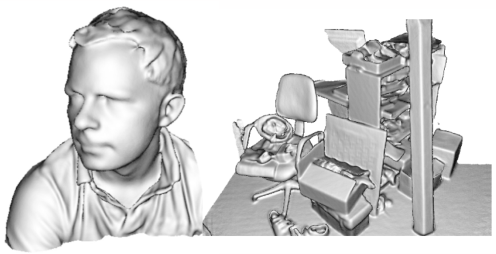
Pottmann, Huang, Yang, Hu: **Geometry and convergence analysis of algorithms for registration of 3D shapes**, IJCV'11

DEMO

Rigid Scanning - Accumulation

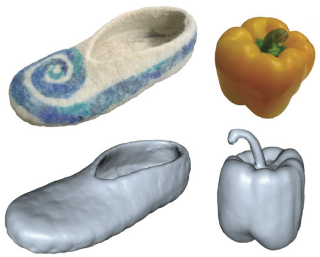
- Reduce the number of points (potentially up to $9x10^6$ V/s of data in VGA)
- Reduce noise (using already collected data)

Volume



Newcombe, Izadi, Hilliges, Molyneaux, Kim, Davison, Kohli, Shotton, Hodges, Fitzgibbon: **KinectFusion: Real-Time Dense Surface Mapping and Tracking**, *ISMAR 2011*

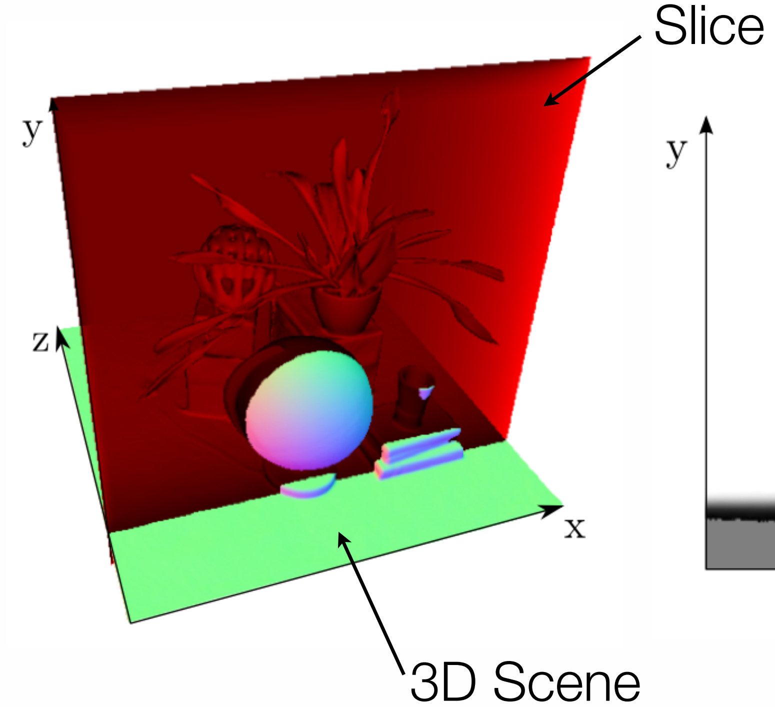
Surface

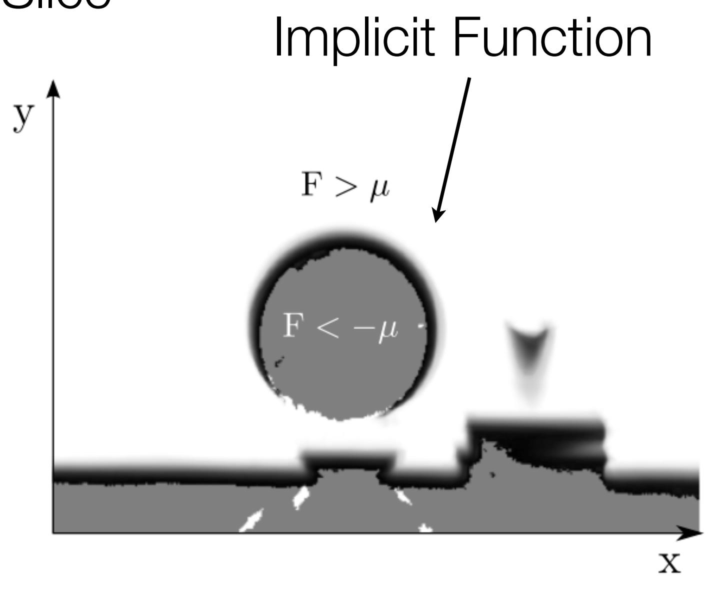


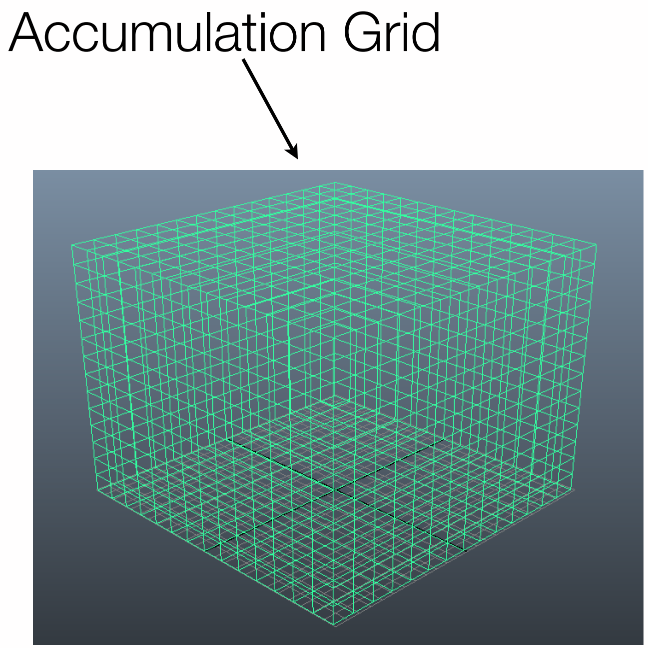
Weise, Wismer, Leibe, Van Gool: **In-hand Scanning with Online Loop Closure**, *3DIM 2009*

Rigid Scanning - Accumulation

Volumetric accumulation







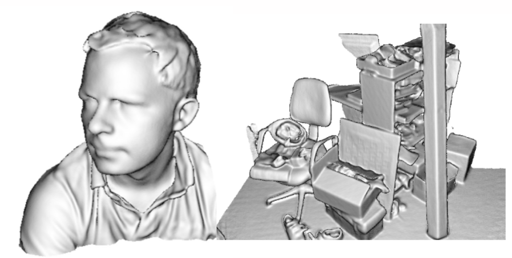
DEMO

Newcombe, Izadi, Hilliges, Molyneaux, Kim, Davison, Kohli, Shotton, Hodges, Andrew Fitzgibbon
KinectFusion: Real-Time Dense Surface Mapping and Tracking, *ISMAR 2011*

Rigid Scanning - Accumulation

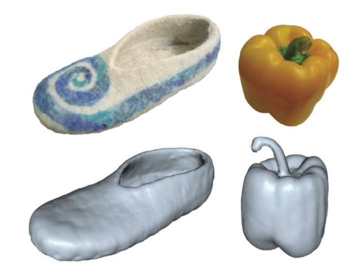
- Reduce the number of points (potentially up to 9×10^6 V/s of data in VGA)
- Reduce noise (using already collected data)

Volume



Newcombe, Izadi, Hilliges, Molyneaux, Kim,
Davison, Kohli, Shotton, Hodges, Fitzgibbon:
KinectFusion: Real-Time Dense Surface Mapping and Tracking,
ISMAR 2011

Surface



Weise, Wismer, Leibe, Van Gool:
In-hand Scanning with Online Loop Closure,
3DIM 2009

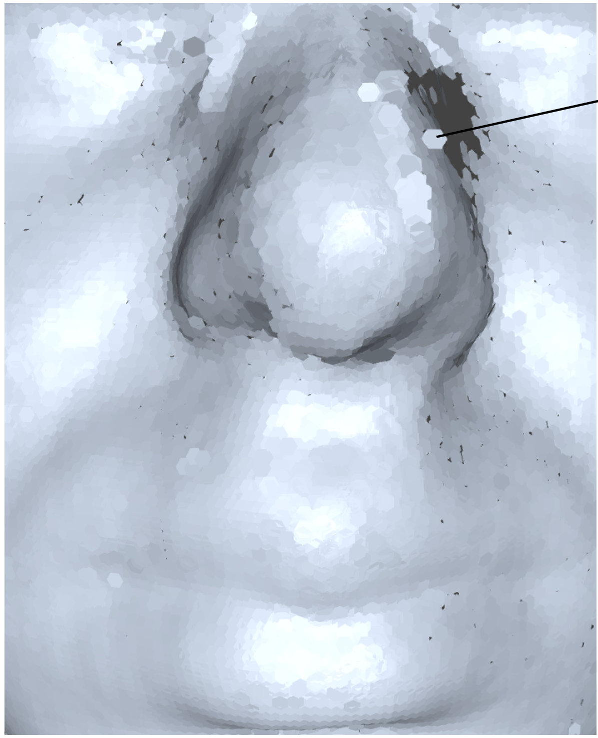
Rigid Scanning - Accumulation

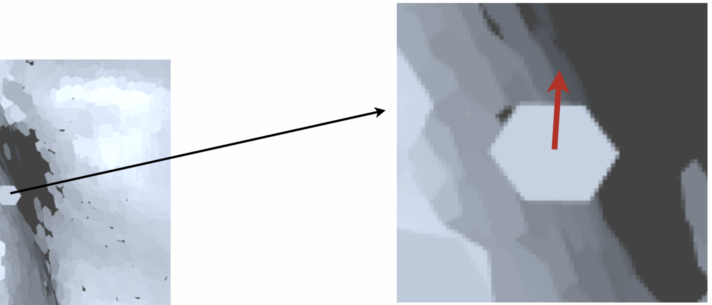
Surface accumulation

Surfel update

Surfel addition

Surfel removal





DEMO

Weise, Wismer, Leibe, Van Gool:
In-hand Scanning with Online Loop Closure,
3DIM 2009

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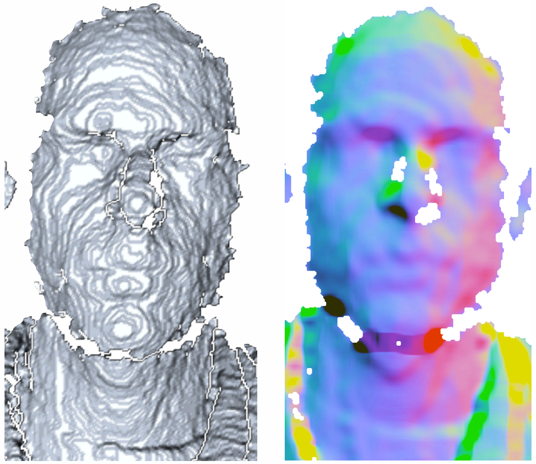
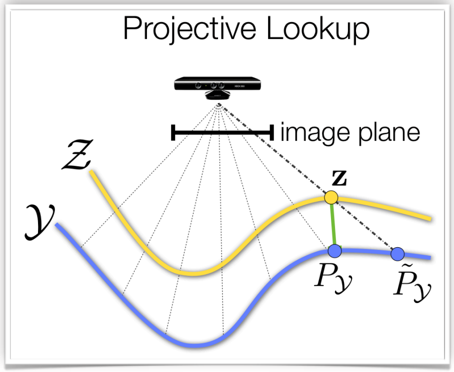
Rigid Scanning - Recap

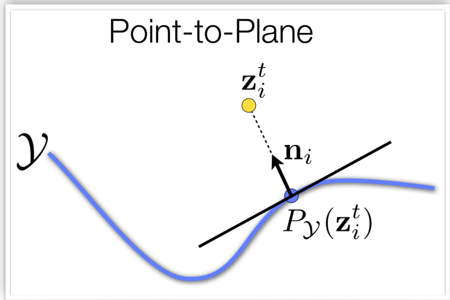
Preprocessing

$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d(i,j) \\ j \cdot d(i,j) \\ d(i,j) \end{bmatrix}$$

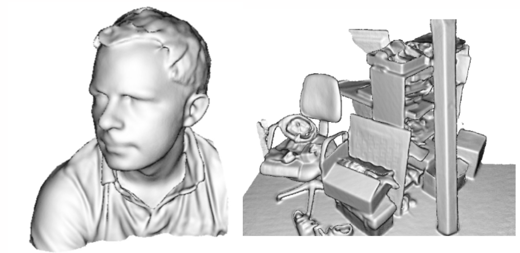


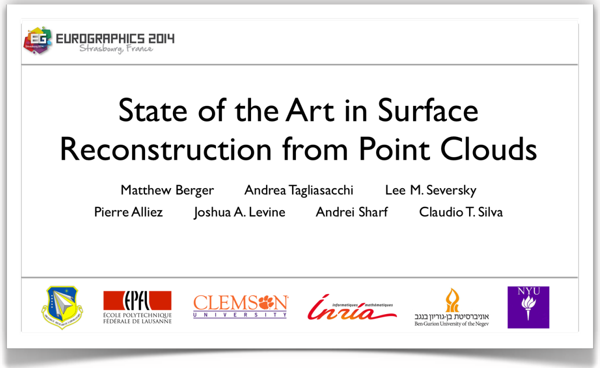
Registration





Accumulation (reconstruction)




EUROGRAPHICS 2014
Strasbourg, France

State of the Art in Surface Reconstruction from Point Clouds

Matthew Berger, Andrea Tagliasacchi, Lee M. Seversky, Pierre Alliez, Joshua A. Levine, Andrei Sharf, Claudio T. Silva

Overview

Introduction (5min)

- Registration (5 min)
  - 3D Geometry (25 min)
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  - Robust Registration (15 min)
  - Q&A (5min)
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  - Articulated Tracking (10 min)
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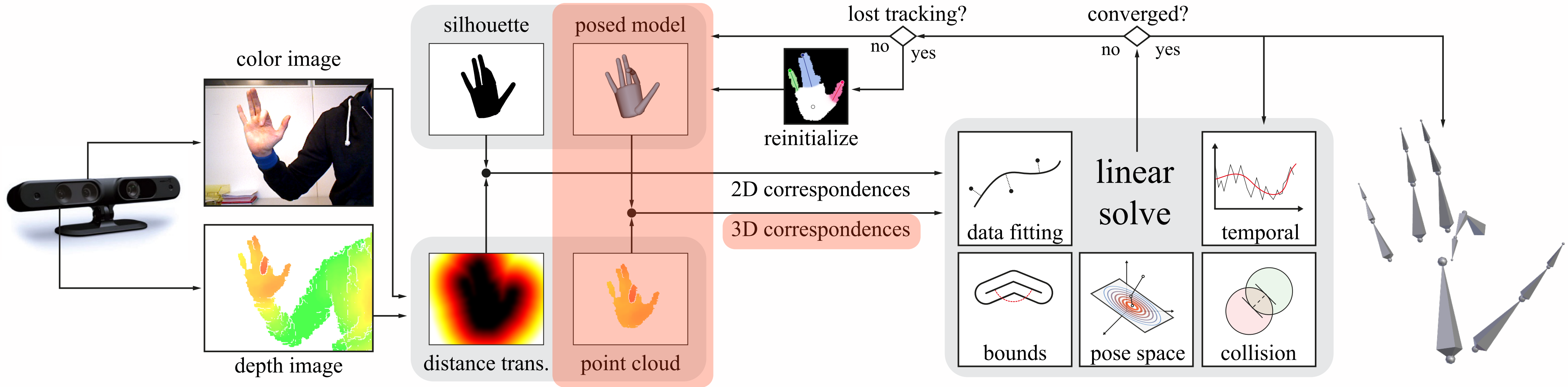
  - Q&A (5min)

Outlook (5 min)

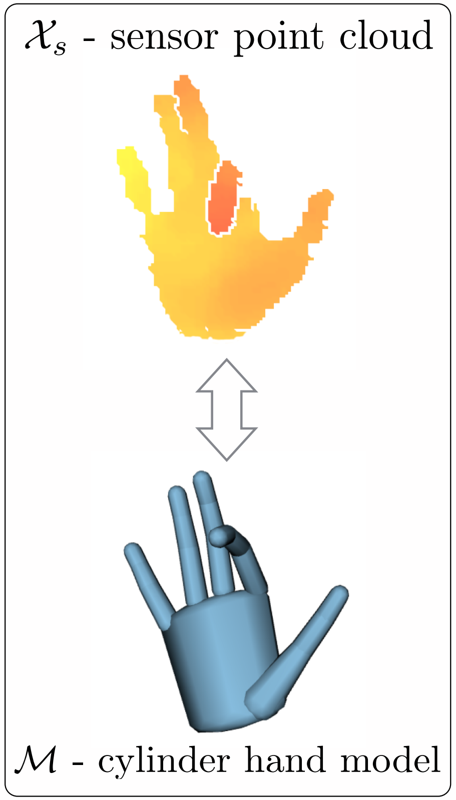
Articulated Tracking

Tiny part of tomorrow's talk: "*Robust Articulated ICP for Real-Time Hand Tracking*"



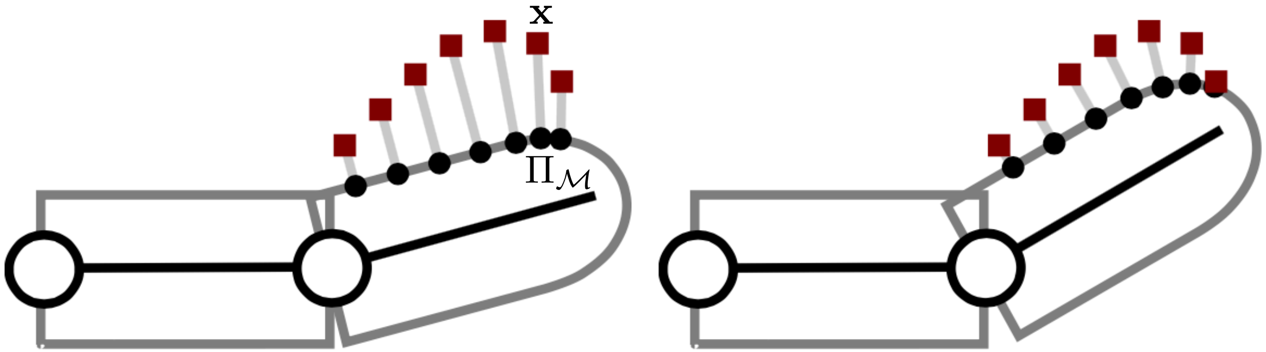
$$\min_{\theta} \underbrace{E_{\text{points}} + E_{\text{silh.}} + E_{\text{wrist}}}_{\text{Fitting terms}} + \underbrace{E_{\text{pose}} + E_{\text{kin.}} + E_{\text{temporal}}}_{\text{Prior terms}}$$

Articulated Tracking

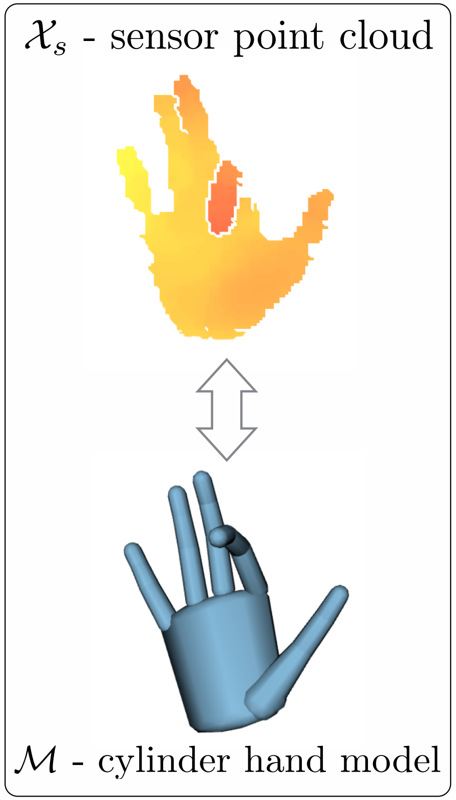


3D Registration

$$E_{\text{points}} = \sum_{\mathbf{x} \in \mathcal{X}_s} \|\mathbf{x} - \Pi_{\mathcal{M}}(\mathbf{x}, \boldsymbol{\theta})\|_2^2$$



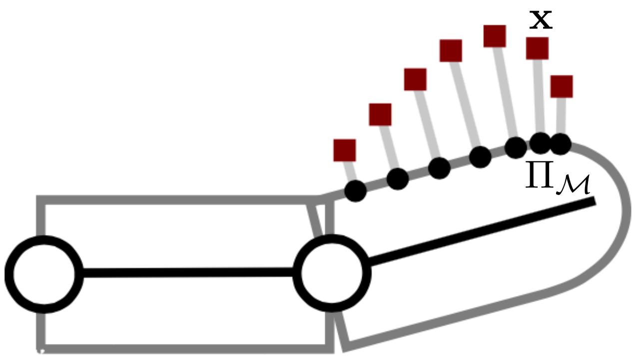
Articulated Tracking

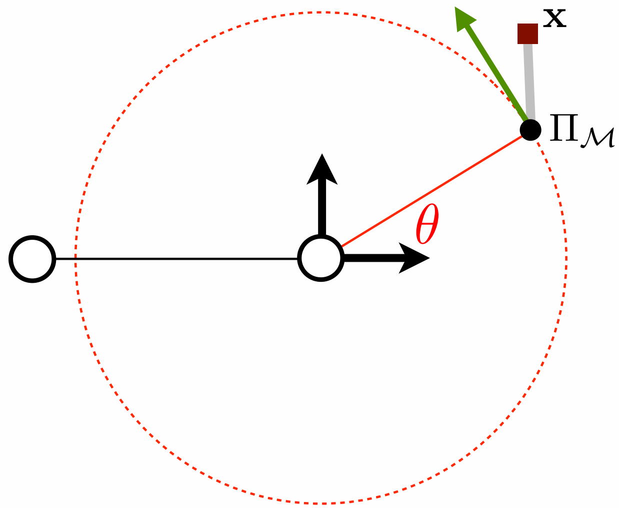


3D Registration

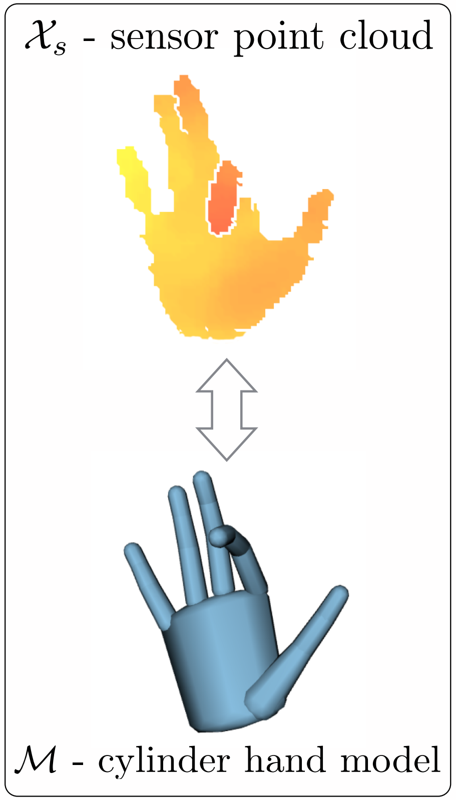
linearize trajectory of closest point corresp.

$$E_{\text{points}} = \sum_{\mathbf{x} \in \mathcal{X}_s} \|\mathbf{x} - \Pi_{\mathcal{M}}(\mathbf{x}, \boldsymbol{\theta})\|_2^2$$



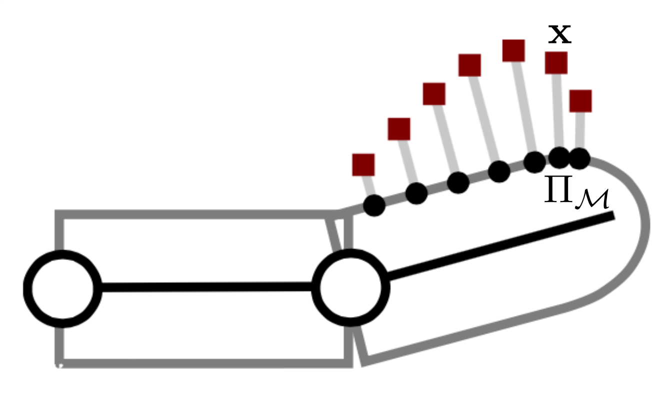


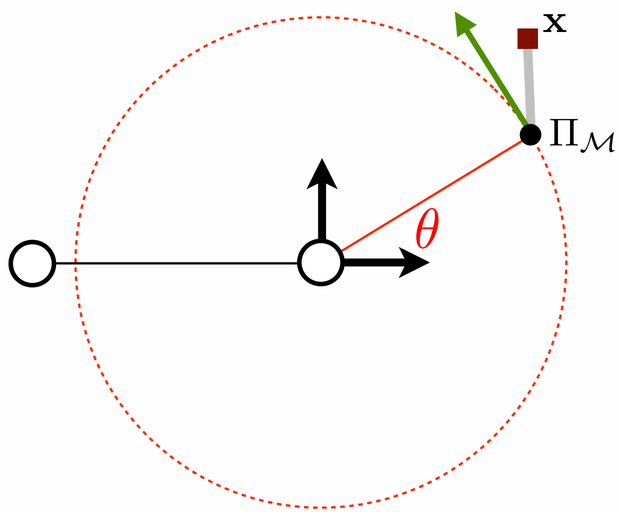
Articulated Tracking



3D Registration

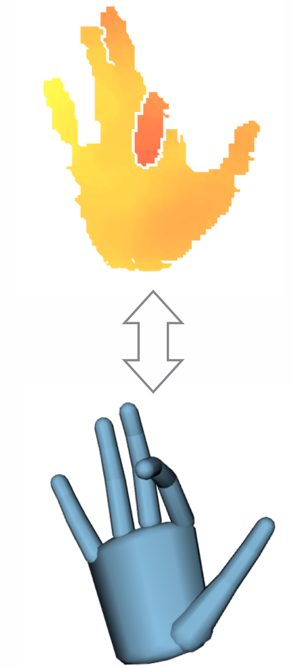
$$E_{\text{points}}(\delta\theta) \approx \sum_{\mathbf{x}\in\mathcal{X}_s} \|\mathbf{J}_{\text{skel}}\delta\theta - (\Pi_{\mathcal{M}}(\mathbf{x},\theta) - \mathbf{x})\|_2^2$$





Articulated Tracking

$\mathcal{X}_s$ - sensor point cloud



$\mathcal{M}$ - cylinder hand model

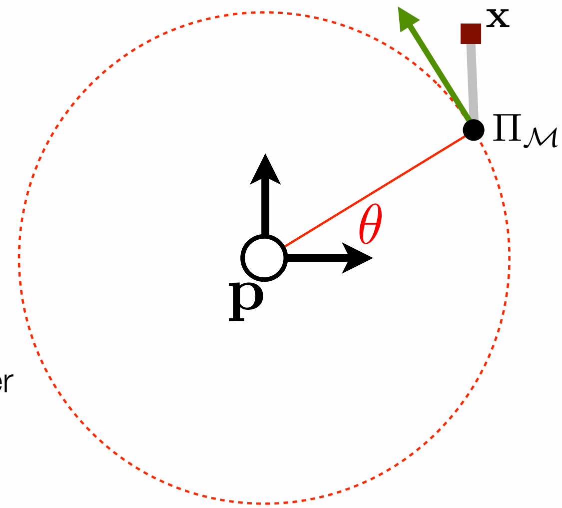
3D Registration

$$E_{\text{points}}(\delta\theta) \approx \sum_{\mathbf{x}\in\mathcal{X}_s} \|\mathbf{J}_{\text{skel}}\delta\theta - (\Pi_{\mathcal{M}}(\mathbf{x},\theta) - \mathbf{x})\|_2^2$$

$$\begin{aligned}\mathbf{J}_{\text{skel}} &= \frac{\partial \Pi_{\mathcal{M}}(\mathbf{x},\theta)}{\partial\theta} \\ &= \mathbf{v} \times (\Pi_{\mathcal{M}}(\mathbf{x},\theta) - \mathbf{p})\end{aligned}$$

joint rotation axis
i.e. [0,0,1]

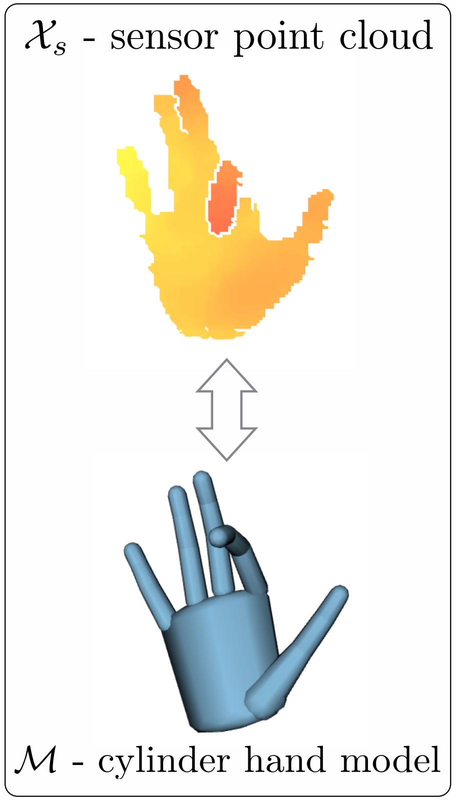
joint rotation center



[Buss'09] Introduction to Inverse Kinematics

Articulated Tracking

[Buss'09] Introduction to Inverse Kinematics

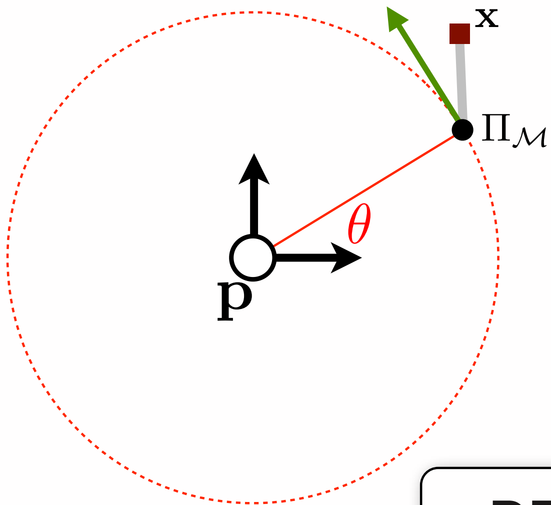


3D Registration

$$E_{\text{points}}(\delta\theta) \approx \sum_{\mathbf{x}\in\mathcal{X}_s} \|\mathbf{J}_{\text{skel}}\delta\theta - (\Pi_{\mathcal{M}}(\mathbf{x},\theta) - \mathbf{x})\|_2^2$$

$$E_{\text{damping}}(\delta\theta) = \|\delta\theta\|_2^2$$

small angle approximation



DEMO

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FÉDÉRALE DE LAUSANNE

Overview

Introduction (5min)

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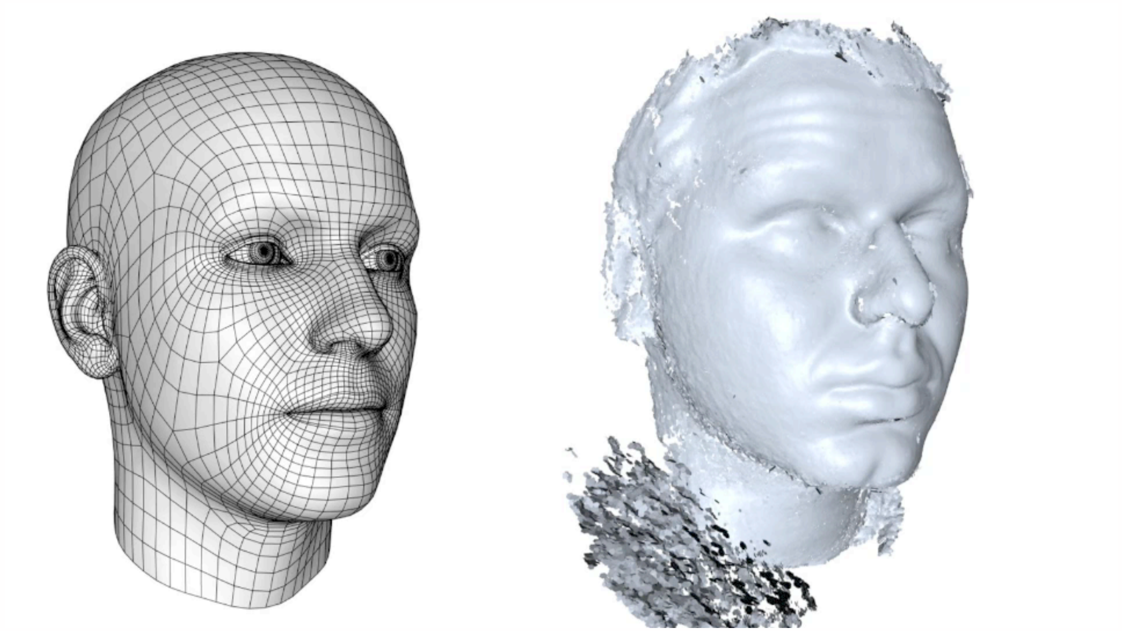
Outlook (5 min)

Non-Rigid Modeling



Non-Rigid Modeling

$$\arg\min_{Z,\{\mathbf{R}_i\}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

matching term

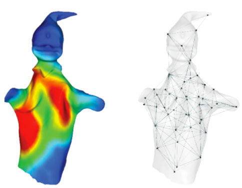
non-rigid prior

Non-Rigid Modeling

$$\arg\min_{Z,\{\mathbf{R}_i\}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

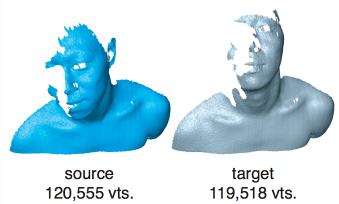
- How to set the weight for the non-rigid energies?
  - Rigid -----> Non-rigid

DEMO



Li, Adams, Guibas, Pauly:
Robust Single-View Geometry and Motion Reconstruction,
ACM SIGGRAPH Asia 2009

Li, Sumner, Pauly:
Global Correspondence Optimization for Non-Rigid Registration of Depth Scans,
SGP 2008



ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Non-Rigid Modeling

$$\arg\min_{Z,\{\mathbf{R}_i\}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

- ARAP prior: geometric energy with one rotation matrix per vertex
- *How can we reduce the number of unknowns?*
  - Data-Driven: Statistical model (PCA)
  - Physical-Based: Modal analysis of deformation energies

Non-Rigid Modeling

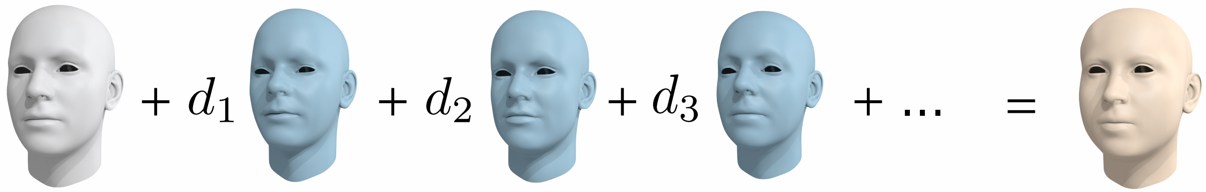
- PCA model

coefficients

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

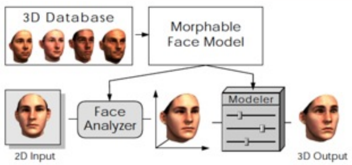
basis

mean shape



$$|\mathbf{d}| << |\mathcal{V}|$$

50 7400



Blanz, Vetter: **A Morphable Model for the Synthesis of 3D Faces**
ACM SIGGRAPH 1999

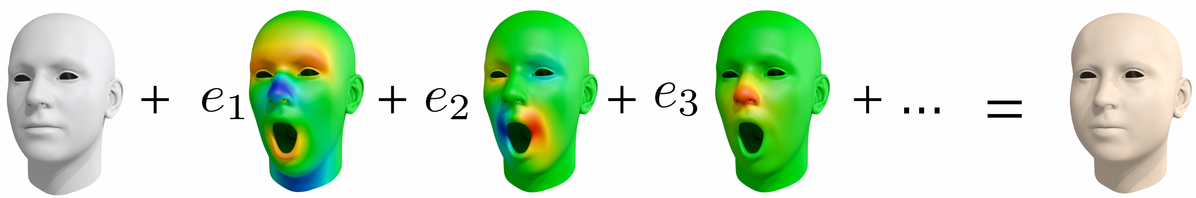
Non-Rigid Modeling

- Modal Analysis (subspace optimization)
  - Assume smooth deformation (no high frequency)
  - Eigenmodes of the Hessian of the deformation energy

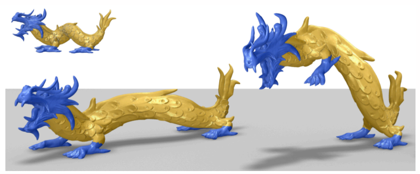
coefficients

$$\mathbf{s} = \mathbf{B}\mathbf{e} + \mathbf{m}$$

basis   base shape

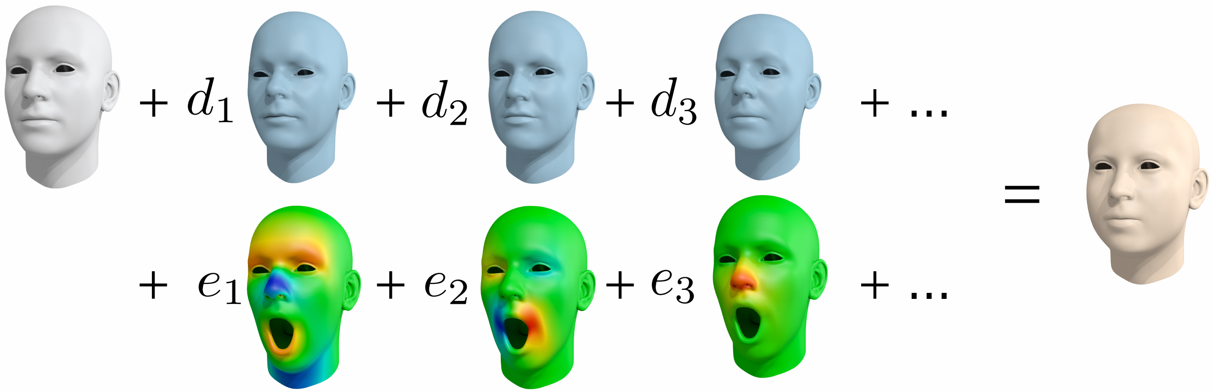


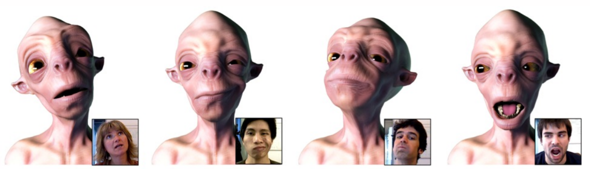
Hildebrandt, Schulz, Tycowicz, Polthier:
Interactive Surface Modeling using Modal Analysis,
ACM SIGGRAPH 2012



Non-Rigid Modeling

- Combining PCA with Modal Analysis

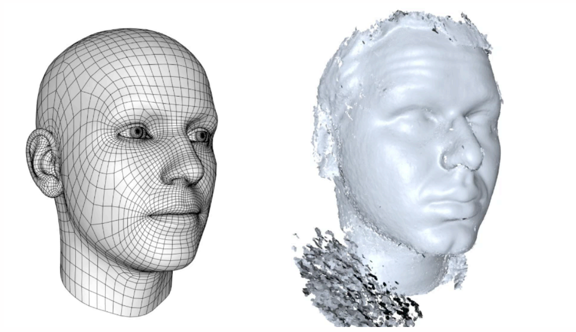




Bouaziz, Wang, Pauly:
Online Modeling For Realtime Facial Animation,
ACM SIGGRAPH 2013

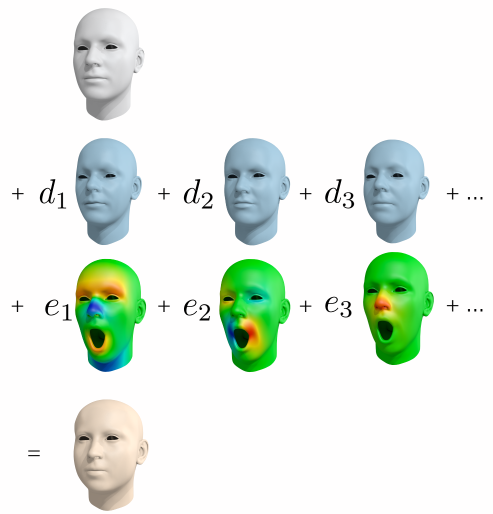
Non-Rigid Modeling - Recap

Registration



$$\arg\min_{Z,\{\mathbf{R}_i\}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$
$$+ w_2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

Advanced Prior



1) Data-Driven: Statistical model (PCA)
2) Physical-Based: Modal analysis

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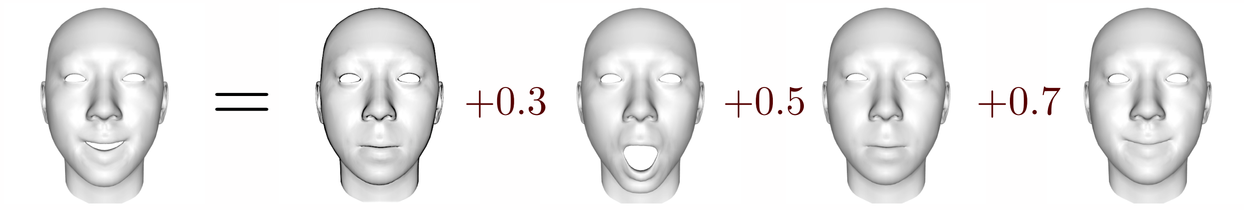
Outlook (5 min)

Face Tracking

- Blendshape model



basis

neutral

$$\mathbf{e} = \mathbf{B}\mathbf{d} + \mathbf{b}$$

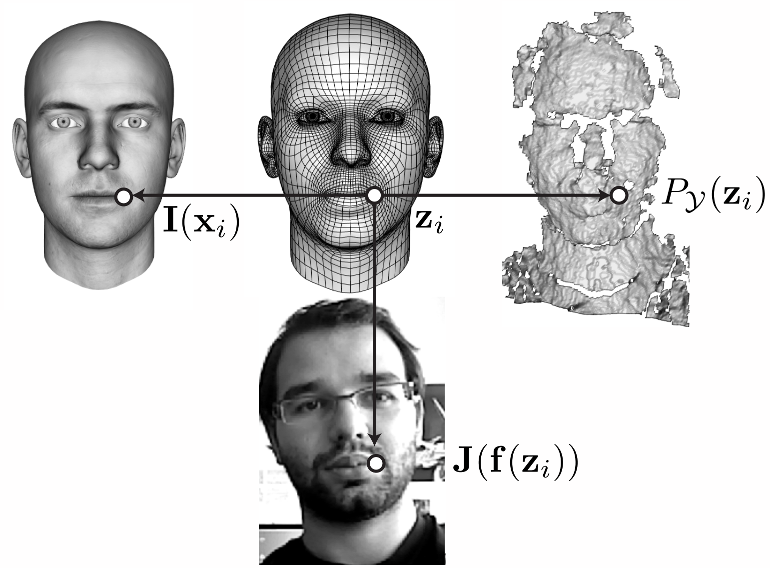
novel expression

coefficients

Face Tracking

$$\arg\min_{Z,\mathbf{d},\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i\mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$

3D matching term — 2D matching term — blendshapes-rigid motion



DEMO

Face Tracking

$$\arg\min_{Z,\mathbf{d},\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i\mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$

3D matching term — 2D matching term — blendshapes/rigid motion

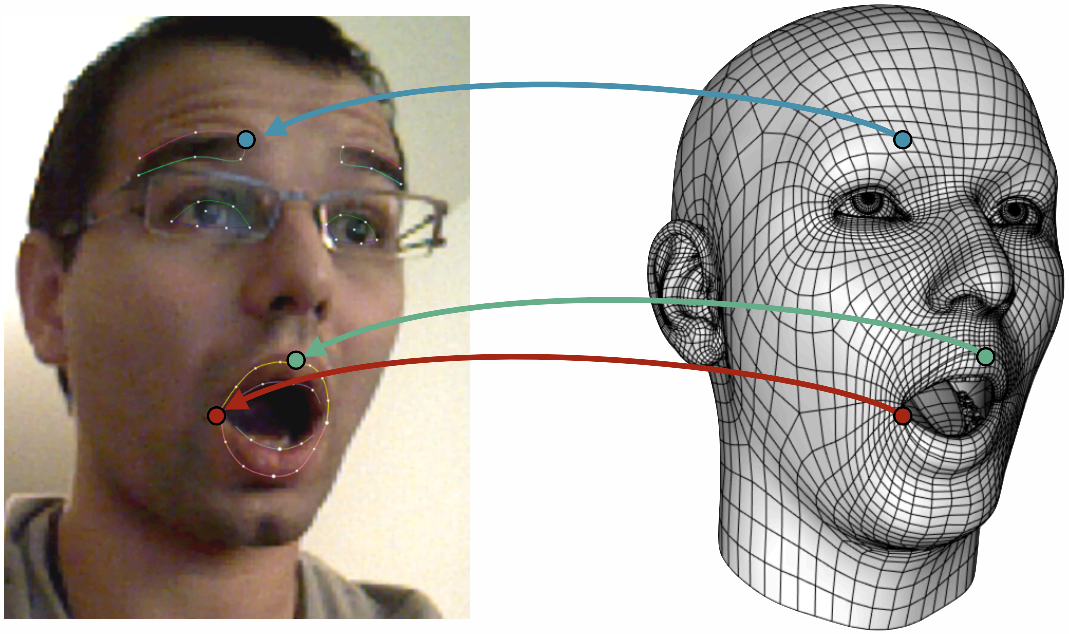
- **Step 1:** find correspondences and optimize rigid motion.
- **Step 2:** optimize blendshape coefficients.

⟷

Iterate until convergence

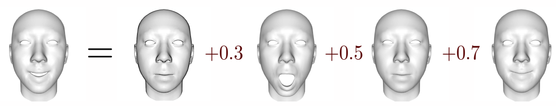
Face Tracking

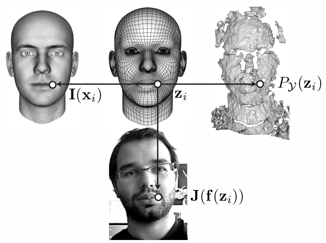
- Until now - registration using local optimization
- Other possibility - alignment using descriptors and features



Face Tracking - Recap

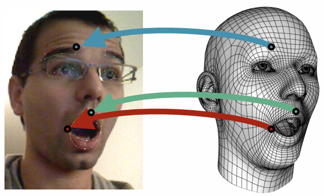
Registration





Step 1: optimize rigid motion

Step 2: optimize coefficients



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Outlook (5 min)

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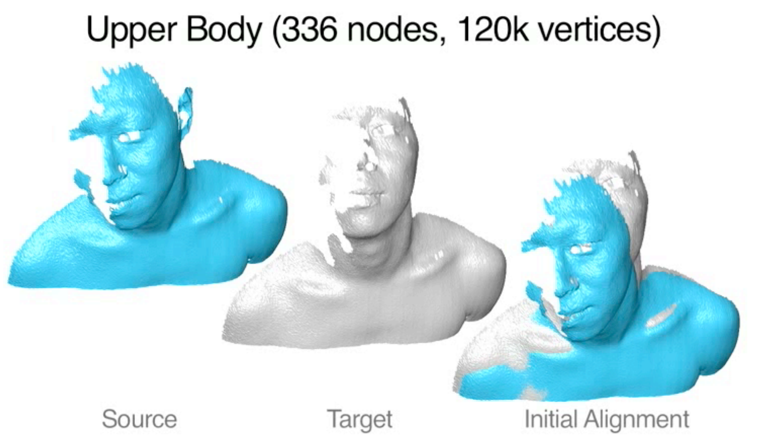
Outlook

- Registration as energy minimization
  - general framework for integrating image and geometry registration
  - matching cost and priors need to be adapted to input data
  - fundamental challenge: correspondences

Outlook

- Registration as energy minimization
  - general framework for integrating image and geometry registration
  - matching cost and priors need to be adapted to input data
  - fundamental challenge: correspondences



Li, Sumner, Pauly, **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

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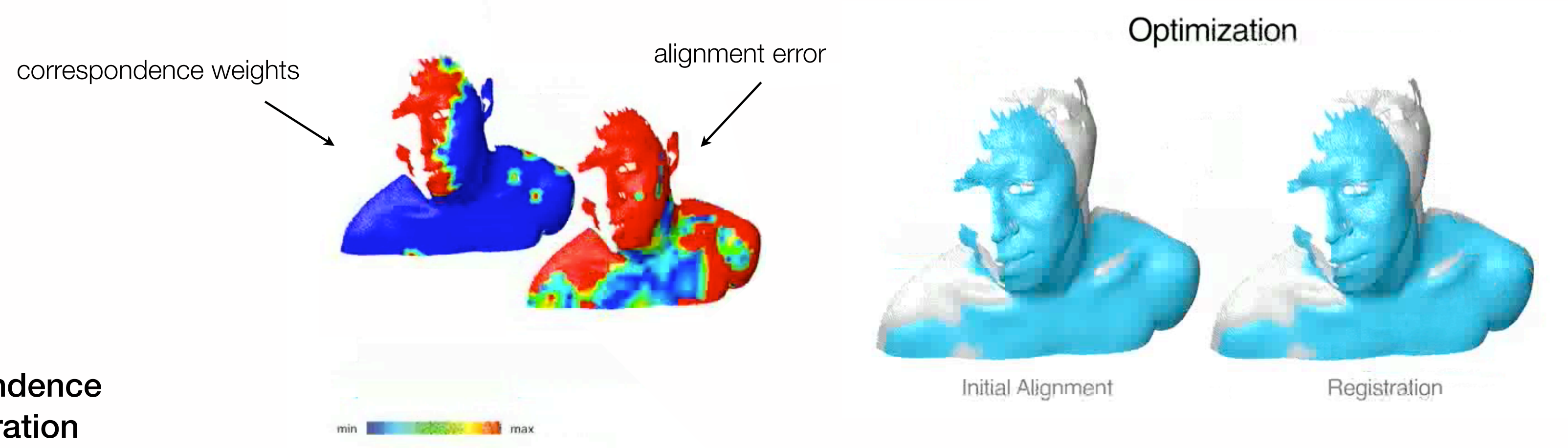

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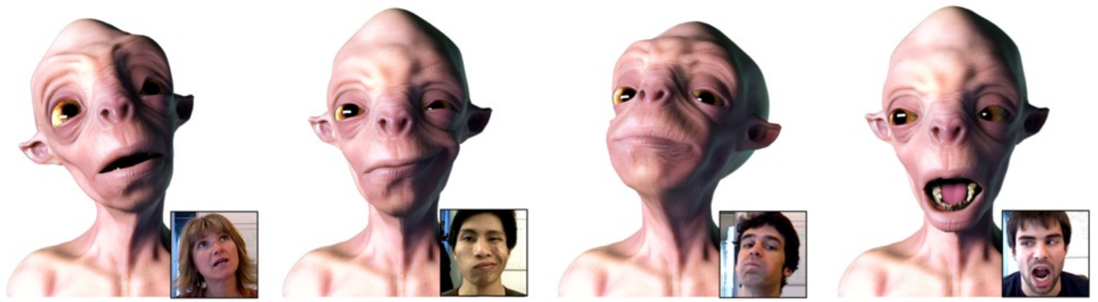
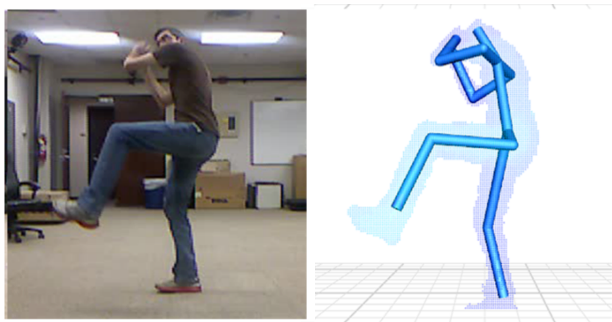
  - fundamental challenge: correspondences
  - can the priors be learned?



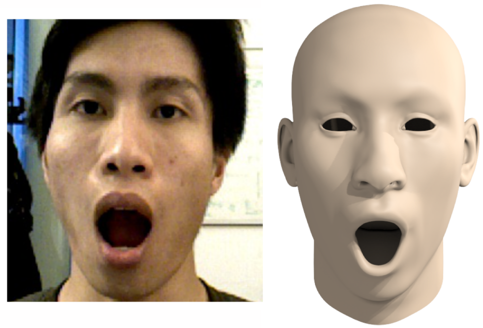
Bouaziz, Wang, Pauly: **Online Modeling For Realtime Facial Animation**, *SIGGRAPH 2013*

Outlook

[Newcombe et al. 2011]



[Wei et al. 2012]



[Bouaziz et al. 2013]

[Li et al. 2013]

[Schroder et al. 2014]

[Chen et al. 2013]

Outlook

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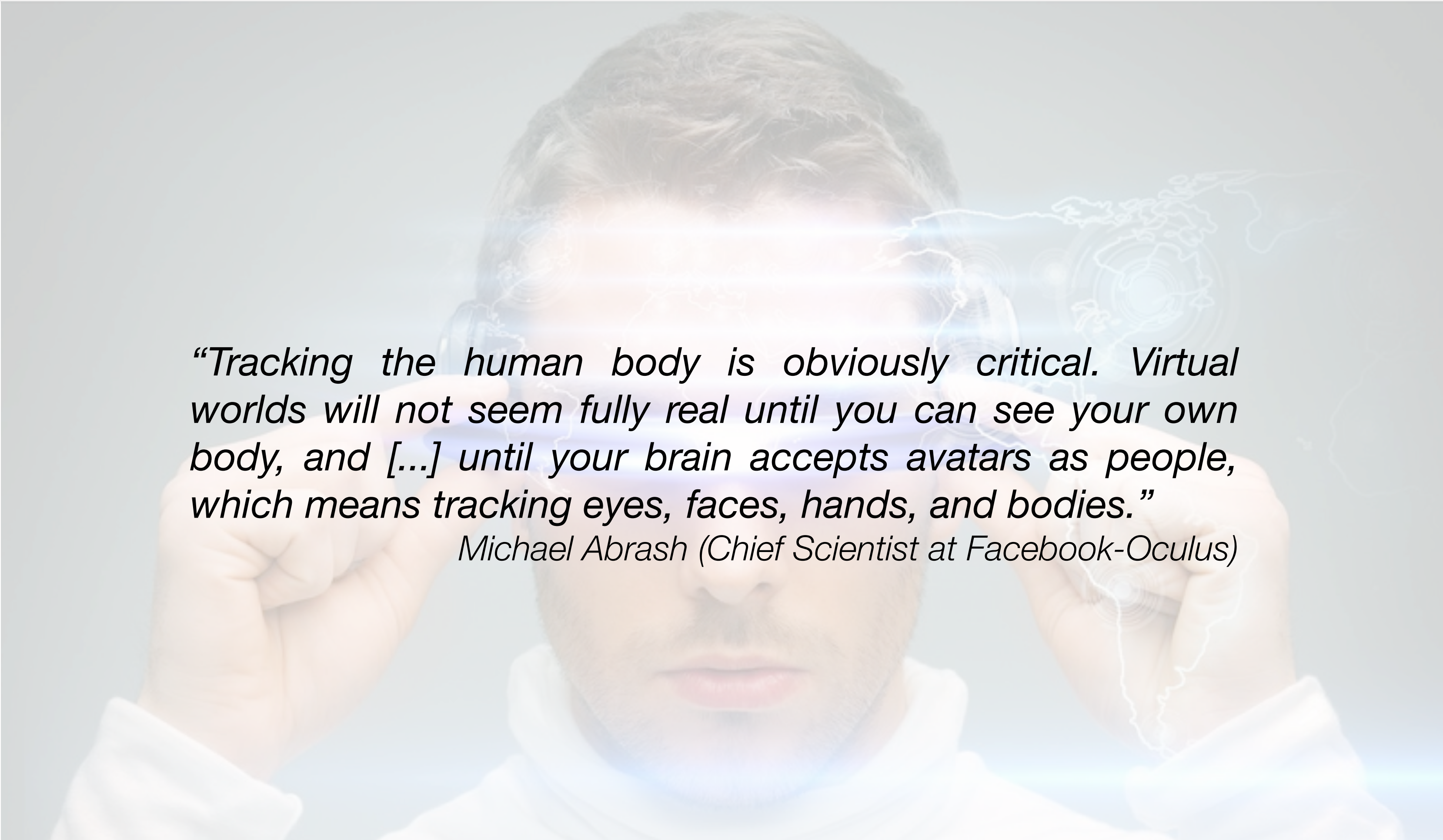

"Tracking the human body is obviously critical. Virtual worlds will not seem fully real until you can see your own body, and [...] until your brain accepts avatars as people, which means tracking eyes, faces, hands, and bodies."

Michael Abrash (Chief Scientist at Facebook-Oculus)

Position Opening / Shameless Advertisement

Who? Prof. Brian Wyvill and Prof. Andrea Tagliasacchi

What? MSc (... PhD)

Where? University of Victoria

Who pays? Google

Language? English

https://www.csc.uvic.ca/Program_Information/Graduate_Studies/msc_program.htm

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Position Opening

Topic: Augmented Reality

Real Time
- Modeling
- Registration
- Physics

Send CV to: andrea.tagliasacchi@gmail.com

Thanks!!!

http://lgg.epfl.ch/publications/2014/2d3dRegistration/

(source code, slides, course notes)

https://github.com/OpenGP/htrack

(real time articulated tracking library)